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Spring 5-28-2021

Meeting their Needs: The Algebraic Knowledge and Instructional Preferences of Students with Learning Disabilities

Kayla Neill
CUNY Hunter College

MEETING THEIR NEEDS: THE ALGEBRAIC KNOWLEDGE AND INSTRUCTIONAL PREFERENCES OF STUDENTS WITH LEARNING DISABILITIES

by

KAYLA NEILL

A dissertation submitted to the Graduate Faculty in Instructional Leadership in partial fulfillment of the requirements for the degree of Doctor of Education, Hunter College

2021

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This manuscript has been read and accepted for the Graduate Faculty in Instructional Leadership in satisfaction of the dissertation requirement for the degree of Doctor of Education.

Date	Nicora Placa
	Chair of Committee
Date	Jennifer Samson
	Supervisory Committee
Date	Melinda Snodgrass
	Supervisory Committee

HUNTER COLLEGE

ACKNOWLEDGEMENTS

First, thank you to the students that participated in this study. It was truly a pleasure learning from you. Your thoughts, wisdom, and knowledge continued to inspire me throughout this journey.

Thank you to my advisor, Nicora Placa, who continued to push my thinking while simultaneously taking on the role of one of my biggest cheerleaders. Thank you, Jen Samson and Melinda Snodgrass, for your support, thoughtfulness, and time. Words cannot express how appreciative I was to have the three of you behind me during this whole process.

To my family, thank you for your continued love and support, and more importantly, for always making me laugh. Thank you for teaching me the value of hard work and dedication, but more importantly, thank you for constantly reminding me to enjoy each and every experience, this one included.

To my Cohort 1 colleagues, friends, and IS 190 family, thank you for listening to me talk about my research, for accepting the fact that I have been exhausted day in and day out for what feels like forever, and for always lending a hand whenever I needed it. I am lucky that I (still) have you all in my life.

DEDICATION

To my family.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Abstract

Meeting their Needs: The Algebraic Knowledge and Instructional Preferences of Students with Learning Disabilities

by

Kayla Neill

Advisor: Nicora Placa

Students with a learning disability (LD) are experts in their lived experiences within the classroom. Yet, little is known about the ways in which students with LD perceive their instruction in mathematics and whether this instruction meets their learning needs. Similarly, the mathematical thinking and content knowledge of students with LD is often excluded from the literature, particularly concerning algebraic concepts. Six high school students with LD from a large urban school district in the northeast United States participated in this two-part study. Semi-structured interviews were used to explore participants' perceptions about their instruction within Integrated Co-Taught (ICT) mathematics classes. By using in vivo coding, data from semi-structured interviews were analyzed across and within cases. Findings from semi-structured interviews are presented in regard to the following four themes: (a) breaking down content, (b) pacing, (c) ensuring student understanding, and (d) group work. Additionally, mathematical task interviews were used to give voice to participants' knowledge of linear functions. Data from mathematical task interview transcripts were analyzed using provisional coding, and data from students' work were analyzed based on its validity and accuracy in relation to the mathematical task. Across tasks, participants demonstrated that their understanding fell into one of the following categories: (a) emergent understanding, (b) procedural understanding, and (c) conceptual understanding. Results are discussed in relation to teacher implications and future

research with the hope that privileging the experiences of students with LD will further improve mathematics instruction for students with LD within an ICT setting.

CHAPTER I

THE PROBLEM

Over the last twenty years, there has been a systematic effort to improve mathematics education in the United States. In addition to standardizing and defining academic proficiency in mathematics across states, the Common Core State Standards for Mathematics (CCSSM) were developed to prepare and promote college and career readiness for all students (NCTM, 2014). Through the design and implementation of coherent mathematics curricula, the CCSSM increase academic rigor and focus on the development of students' conceptual understanding. Even though the Individuals with Disabilities Education Act (IDEA; 2004) mandates schools to provide students with a learning disability (LD) access to the same curriculum and CCSSM as general education students (Cramer, 2015; Jitendra, 2013; Lambert, 2018), students with LD may struggle academically compared to their peers without disabilities in mathematics (Cortiella & Horowitz, 2014). In addition to earning lower grades on standardized tests such as the National Assessment of Educational Progress (NAEP), and in class, as a group, students with LD are more likely to experience higher rates of course failure compared to students without disabilities (Cortiella & Horowitz, 2014). Furthermore, in the United States, only 68% of students with LD receive a regular high school diploma (Cortiella & Horowitz, 2014) compared to approximately 85% of all students (National Center for Education Statistics, 2019).

For students with LD to succeed at higher levels of mathematics, they must develop algebraic reasoning (Kaput, 1998; Thompson & Carlson, 2017). Linear functions is a fundamental algebraic concept (Hitt, 1998; Oehrtman et al., 2008; Teuscher & Reys, 2010; Wang et al., 2017) that is typically introduced in middle school, and it continues to appear throughout advanced mathematics classes (Teuscher & Reys, 2010). The concept of a linear function is presented to students as a dependent relationship typically expressed as $y = mx + b$, where m represents the rate of change and b represents the y-intercept. A linear function may also be represented as a straight line on a coordinate plane with a constant rate of change (Wang et al., 2017). Dubinsky (1993) stated, “It can be argued that functions form the single most important idea in all mathematics, at least in terms of understanding the subject as well as for using it" (p. 527). However, researchers have found that students, including students with LD, often experience great difficulty learning algebraic concepts such as linear functions (Davis, 2007; Herbert & Pierce, 2012; Knuth, 2000; Teuscher & Reys, 2010; Wilkie & Ayalon, 2018).

Issues in developing their conceptual understanding in algebra may arise for students with LD because of the level of abstract thinking that algebra entails (Witzel et al., 2003). Rather than portraying mathematics in pictures or using concrete representations, algebra requires students to recognize and manipulate symbols as well as understand numerical relationships and mathematical structures (Linsell, 2009). Students often rely on memorizing facts and procedures (Capraro & Joffrion, 2006), or they resort to utilizing guess-and-check methods (Herscovics & Linchevski, 1994). Both of these strategies tend to be problematic for students with LD because they are time-consuming and depend on students’ working memory. In addition to having trouble with their working and long-term memory, students with LD experience difficulties organizing and manipulating information (Jitendra, 2013). As a result, students may struggle to develop complex algebraic reasoning that would help them solve difficult tasks (Capraro & Joffrion, 2006; Knuth et al., 2005). Additionally, teachers may limit students with LD from developing a deep understanding of algebraic concepts because of the manner in which they introduce mathematical content. Often teachers of students with LD utilize explicit instruction, which focuses on the memorization of procedures or the use of heuristics (Gersten et al., 2009; Lambert, 2018; Powell et al., 2013; Watt et al., 2016). As a result, students taught to use procedural approaches grapple with applying algebraic skills to different situations (Capraro & Joffrion, 2006; Ellis, 2007).

While proven as an effective teaching practice for students with LD (Gersten et al., 2009; Hattie, 2009), the use of explicit instruction conflicts with the CCSSM push for the active construction of knowledge through inquiry-based exploration, discussion, and reflection. This clash in instructional pedagogy (explicit versus inquiry-based) may create a potential challenge for teachers because there has been an increase in the number of students with LD educated in a general education setting for 80% or more of their school day (Cortiella & Horowitz, 2014). In the United States, co-teaching is a popular model used to educate students with LD in a general education setting (Cook et al., 2017). Co-teaching, also known as an Integrated Co-Taught (ICT) class, is a classroom setting in which a content teacher and a special education teacher work together as co-teachers to provide instruction to general and special education students (Cook & McDuffie-Landrum, 2020; Murawski & Lochner, 2011). Overall, the premise of an ICT class is that both co-teachers share the responsibility of implementing the standards-based curriculum to all students, including students with LD. However, the ways that co-teachers implement instruction within an ICT class may vary from school to school (Rexroat-Frazier & Chamberlin, 2019).

Little is known about the mathematical thinking of students with LD on linear functions or their perceptions of the instructional approaches used by co-teachers that best support their learning. For example, Lambert and Sugita (2016) found only seven qualitative peer-reviewed studies that showed evidence of the inclusion of students with disabilities in classrooms implementing a standards-based mathematics curriculum. Within those studies, student engagement differed vastly, and none of the studies included high school students (Lambert &

Sugita, 2016). As most of the research on students with LD in mathematics is quantitative in nature (Gersten et al., 2009; Lambert & Tan, 2017; Watt et al., 2016), the mathematical thinking of students with LD and their perceptions of their mathematics education have been left out of the literature (Lambert & Tan, 2017). Due to the limited research on the participation of students with LD in standards-based mathematics classes, it is unknown whether they can access the curriculum, demonstrate mastery of grade-level mathematical practices and standards, and develop their conceptual understanding. Rather, if progress is to be made in improving the algebraic skills of students with LD, there is a need to know more about their thinking and the ways in which they perceive their mathematics instruction.

Research Questions

1. Within an ICT setting, what types of instructional practices do students with LD perceive as supportive for their success in mathematics?
2. What conceptions of linear functions do students with LD possess, as evident in their work on problems with abstract graphical representations and real-world connections?
3. Based on existing literature, to what extent, if any, does the way in which students with LD approach tasks on linear functions differ from students without disability labels?

Theoretical Frameworks

In this study, the researcher sought to explore the preferences of students with LD regarding the instructional practices used within their ICT mathematics classes and their content knowledge of linear functions. As such, both the sociocultural theory and the constructivist theory were used to frame this study.

Sociocultural Theory

In this study, the sociocultural theory was used in an effort to understand the ways in which students with LD perceive their instruction in mathematics within an ICT setting. Within this theory, students are thought to develop mathematical knowledge and skills through social interactions with those who hold a deeper level of understanding, such as teachers or peers. To encourage the development of mathematical knowledge, students need to participate in learning experiences through communicating with others to create a shared meaning (Steele, 2001). By communicating with others, students' "growth of mathematical understanding occurs through a process of connecting earlier thought with new mathematical language in order to create more meaning. Explaining one's thoughts to others becomes reasoning for oneself" (Steele, 2001, p. 405). In the classroom, teachers are responsible for creating opportunities for students to appropriate new information through participating in joint activities. Students construct a concrete understanding through their social interactions (Vygotsky, 1978).

In this study, the sociocultural theory was used to analyze students' perceptions of their current and past instruction in mathematics in an ICT setting. Because the sociocultural theory focuses on the interactions mediated through language, this framework provided the opportunity to learn about the types of interactions students with LD found useful for their development of mathematical concepts. For example, how did students with LD value working in small groups with their peers? Was it helpful for students with LD to communicate their thinking to their teachers and peers? In addition, through the lens of the sociocultural theory, the researcher investigated the ways that students perceived their teachers' efforts to incorporate or limit teacher-to-student and student-to-student interactions.

Constructivist Theory

The constructivist theory was used to explore the cognitive development of students with LD. Within the constructivist theory, students use their knowledge to construct new mathematical knowledge (Woodward & Montague, 2002). Piaget (1967) noted, “All knowledge is tied to action, and knowing an object or an event is to use it by assimilating it to an action scheme” (pp. 14-15). By participating in goal-directed activities, students produce results. Either these results assimilate into the mathematical schemas that students have previously solidified, or the results cause stress or disturbance to their existing schema. As a result, students may be required to change the construction of their schemas to accommodate this new knowledge (von Glaserfeld, 1995).

Because the purpose of this study was to explore the mathematical thinking of students with LD on linear functions, the constructivist theory provided a framework for understanding the current schema of linear functions for each student with LD. Throughout the mathematical task interview, students shared their current schema while completing several mathematical tasks. Probing questions were utilized not only to understand participants’ current schema better, but also to elicit information about the ways in which their actions assimilated to their current schema. Additionally, the constructive theory provided a framework to understand how participants tried to make sense of mathematical knowledge that did not align with their current schema and their attempt to construct a new schema.

Conceptual Frameworks

The purpose of this study was to explore ways that mathematics education researchers and educators can better support students with LD in mathematics by giving voice to their knowledge and experiences. IDEA defines a specific learning disability as

> A disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, write, spell, or to do mathematical calculations. (IDEA, 2004, Sec. 300.8(c)(10))

Research on ICT mathematics classes, instructional practices used to support students with LD in mathematics, and student voice framed the methods that were used to gather data in this study.

Integrated Co-Teaching

In the 1980s, only 16% of students with LD were included in general education classes (Gartner & Lipsky, 1987). Instead, students with LD were educated separately, typically known as a self-contained class, under the premise that they would benefit from a smaller class with a specialized teacher and materials (Gartner & Lipsky, 1987). However, students in self-contained classes were less likely to receive standards-based instruction and instruction of conceptual strategies compared to general education classes (Jackson & Neel, 2006). Within self-contained classes, the instruction was not as complex as instruction in general education classes (Wehmeyer, 2006). Additionally, students with disabilities were more engaged in academic-focused instruction in general education classes than students that were educated in self-contained settings (Logan & Keefe, 1997). Not only can students with disabilities show higher levels of achievement (Gartner & Lipsky, 1987) and student engagement, but also all students could benefit from the diversity in a classroom of students with and without disabilities.

Federal legislation, such as IDEA and No Child Left Behind (NCLB; 2002), required schools to educate students with LD with their general education peers and provide students access to the general education curriculum. Consequently, schools started to educate students with LD with general education students through a practice known as ICT (Scruggs et al., 2007).

Within an ICT class, a content teacher and a special education teacher work together as co-teachers to provide instruction to general and special education students in a way that meets the learning needs of a diverse group of students (Friend, 2008). Friend et al. (2010) defined co-teaching as

> The partnering of a general education teacher and a special education teacher or another specialist for the purpose of jointly delivering instruction to a diverse group of students, including those with disabilities or other special needs, in a general education setting and in a way that flexibly and deliberately meets their learning needs. (p. 11)

In an ICT setting, Friend et al. (2010) noted that teachers should allow for and encourage students to engage with each other rather than separating students with LD into their own group. Ideally, ICT provides the opportunity for students to learn from one another academically and socially.

When planned and facilitated strategically, all students can benefit from an ICT class (Brendle et al., 2017; Scruggs et al., 2007). The premise of ICT is that instruction will improve because "the expertise of the masters of content – the content area teachers – are blended with and supported by the expertise of the masters of access – the specialists in differentiating instruction" (Villa et al., 2008, p. 16). In theory, co-teachers are better equipped to support students with disabilities and students that are at risk for failure because there are two teachers in the class, each with their own expertise (Cook et al., 2017; Magiera et al., 2005; Sileo & van Garderen, 2010). Co-teachers have access to a wider range of instructional strategies (Cook et al., 2017; Mastropieri et al., 2005), and they are expected to include these instructional and alternative strategies to improve the performance of students with varying abilities in the class (Cook & Friend, 1995). Additionally, co-teachers believe that ICT is advantageous to students because students have more opportunities to receive individualized attention from their teachers (Rice & Zigmond, 2000; Scruggs et al., 2007; Sileo & van Garderen, 2010; Walther-Thomas, 1997). Throughout instruction, special educators can monitor student progress, identify students that are struggling, and pull these students to a small group to provide appropriate and individualized support. In addition to academic benefits, participation in an ICT setting can help students with disabilities increase their self-esteem, confidence, and peer relationship skills (Fontana, 2005; Walther-Thomas, 1997).

Some co-teachers may find it difficult to execute co-teaching models and strategies effectively (Scruggs et al., 2007). As a result, a common model of co-teaching utilized in mathematics classes is *one teach, one assist* (King-Sears & Strogilos, 2018; Pancsofar & Petroff, 2016; Weiss & Lloyd, 2002). Usually, within the *one teach, one assist* model, the general education teacher leads instruction while the special education teacher moves around the classroom to assist students. In their work with elementary school teachers of a mathematics ICT class, Brendle et al. (2017) found that co-teachers believed co-teaching was beneficial for students, but that they lacked expertise in implementing a variety of co-teaching models. As a result, co-teachers relied mostly on the *one teach, one assist* model. Similarly, Magiera et al. (2005) found that co-taught mathematics classes continued to resemble the traditional class format, with the general education teacher primarily utilizing whole-class instruction. Within these classes, special education teachers had to find ways to support students through one-on-one instruction (Magiera et al., 2005). Often other circumstances force co-teachers to utilize the *one teach, one assist* model, such as the special education teacher lacking the content knowledge of secondary mathematics (Weiss & Lloyd, 2002; Zigmond & Matta, 2004). By having two teachers instead of one in the classroom, it would seem that instruction and access to the standards-based curriculum for students with LD would improve. However, two teachers within an ICT setting does not ensure that instruction will differ from traditional, whole-class instruction. As such, research on mathematics co-teaching is needed because “far more literature exists describing co-teaching and offering advice about it than carefully studying it” (Friend et al., 2010, p. 9).

As students within an ICT class are consumers of instruction, research should privilege their lived experiences (Lambert, 2016). However, there is limited research on the perceptions of students with LD about their ICT instruction, and even less is known in terms of their mathematics instruction (Strogilos & King-Sears, 2019). In their work with seventh- and eighth-grade students in reading and language arts, Embury and Kroeger (2012) found that seventh-grade students identified the general education teacher as the main teacher in charge. Students perceived that the special education teacher was responsible for working with students who may not learn as quickly as other students. However, in an eighth-grade class in which co-teachers used a variety of co-teaching models, students shared that both of their teachers were helpful to all students. In terms of middle school mathematics instruction, King-Sears and Strogilos (2018) found that students and co-teachers perceived that the *one teach, one assist* co-teaching model was used most frequently. Similar to Embury and Kroeger’s (2012) work, Strogilos and King-Sears (2019) found that even though students could ask both teachers for help in mathematics, students shared that the general education teacher was the lead instructor and was the one in charge of the lesson. While not focusing mainly on mathematics instruction, in their study of high school students with LD, Leafstedt et al. (2007) found that students preferred to receive their instruction in a resource room setting in which they were separated from the ICT class.

These findings may possibly be due to the lesson's pace, the style of teaching, or the number of students in the class. Leafstedt et al. (2007) noted that

> Students gave a great deal of importance to the special education teacher being able to teach fewer students, change the pace of the lesson, and teach in a different manner within the special education setting. This is in conflict with the rationale for co-teaching, which states that students will receive a wider range of instructional options when in a co-taught classroom. (p. 182)

Existing research on ICT settings framed the semi-structured interview questions in this study with the purpose of learning more about the experiences of students with LD and their perceptions of instructional strategies used within their ICT mathematics classes.

Instructional Practices

To better support students with LD, there is a need to know which teaching practices students find beneficial for their learning. In an attempt to identify effective interventions for students with LD in algebra, Watt et al. (2016) reviewed literature from 1980 to 2014. To be included in their study, research was experimental, quasi-experimental, or a single case study design that included students with LD, focused on an algebraic concept aligned to a CCSSM, and measured the effect of an instructional intervention on student achievement. Only 15 articles met the requirements of their study, five of which were single case designs. Watt et al. used standardized mean differences from each study and created a total of 10 effect sizes, which were scaled to Hedge's *g*. Watt et al. found that the concrete-to-representational-to-abstract (CRA) approach was the most common intervention and produced high effects ($g = 0.53$). All four studies that used the CRA approach found that participants showed significant growth from their pretest to posttest score (Scheurmann et al., 2009; Strickland & Maccini, 2013; Witzel, 2005;

Witzel et al., 2003). In addition, Watt et al. found that peer tutoring (g = 0.40), heuristic or mnemonic devices (g = 0.83), and graphic organizers (g = 0.57) were effective to highly effective strategies.

To synthesize existing research, Gersten et al. (2009) conducted a meta-analysis on mathematics interventions for students with LD. After reviewing over 30 years of research, Gersten et al. included 42 quantitative studies in their analysis. After determining which studies addressed instruction or curriculum design, Gersten et al. coded each of these studies into one of six instructional components. Similar to the work of Watt et al. (2016), Gersten et al. calculated the effect size of each study as Hedge's *g* and found the mean effect size (ES) for each of the six instructional components. In their work, they found that the following components of instruction were beneficial for students with LD: (a) explicit instruction (n = 11, ES = 1.22, $p < .001$); (b) the use of heuristics (n = 4, ES = 1.56, $p < .001$); (c) student verbalizations of their mathematical reasoning (n = 6, ES = 1.04, $p < .001$); (d) visual representations (n = 12, ES = 0.47, $p < .001$); and (e) range and sequence of examples (n = 9, ES = 0.82, $p < .001$). In addition, Gersten et al. (2009) highlighted that teacher feedback (n = 7, ES = 0.21, $p < .10$) and student feedback (n = 7, ES = 0.23, $p < .05$) were effective instructional components. Of these studies, only two focused on mathematics interventions for students with LD with a particular focus on algebraic concepts. Because the work of Watt et al. (2016) and Gersten et al. (2009) focused on studies that were quantitative in nature, little is known about the thoughts of students with LD in regard to these instructional practices. Additionally, many of these studies focused on a small group or individual student interventions. As such, it is unknown whether these instructional practices are used within ICT settings, and if they are used, to what extent students with LD perceive them as useful and advantageous for their learning. Instructional practices identified by Gersten et al.

(2009) and Watt et al. (2016) helped develop semi-structured interview questions in an effort to answer the first research question in regard to the types of instructional practices that students with LD perceive as supportive for their success in mathematics.

Explicit instruction is an instructional practice commonly used to educate and study students with LD in mathematics. While explicit instruction was originally a teaching strategy used to facilitate high-quality instruction to a small group of students (Archer & Hughes, 2011), eventually, teachers began to utilize explicit instruction within a whole-class setting (Doabler & Fien, 2013; Hughes et al., 2017). In their work, Riccomini et al. (2017) defined explicit instruction as

> A group of research-supported instructional behaviors used to design and deliver instruction that provides needed supports for successful learning through clarity of language and purpose, and it promotes active student engagement by requiring frequent and varied responses followed by appropriate affirmative and corrective feedback, and assists long-term retention through use of purposeful practice strategies. (p. 4)

Within explicit instruction, the teacher models and breaks down a new mathematical concept into discrete units or steps (Doabler et al., 2012; Weibe Berry & Namsook, 2008). The idea is that by breaking down content into parts, the teacher reduces the cognitive load placed on students based on their current skills (Archer & Hughes, 2011). Each part of the demonstration is clear and unambiguous (Archer & Hughes, 2011; Doabler & Fien, 2013). Following the completion of the demonstration, the teacher leads students in guided practice, which consists of students working with their teacher to complete a similar mathematical task to the one that the teacher modeled. During this time, students begin to take on some of the responsibility of solving the task (Doabler & Fien, 2013). The teacher can monitor student understanding and provide appropriate and immediate feedback by asking questions and eliciting student participation (Doabler & Fien, 2013; Hughes et al., 2017). Using students' responses as evidence of their understanding, the teacher may even adjust instruction to meet the needs of the students (Heward & Wood, 2013). After guided practice, the teacher invites students to work independently, in pairs, or in small groups, to complete a similar task or series of tasks related to the mathematical concept that was modeled during instruction. By monitoring student progress, the teacher can provide small group support, if needed (Riccomini et al., 2017).

The Council for Exceptional Children and the Collaboration for Effective Educator, Development, Accountability, and Reform (CEEDAR) Center identified explicit instruction as one of 22 "High Leverage Practices" for students with disabilities (McLeskey et al., 2017). Research has shown that students with LD benefit from direct, explicit instruction (Bryant et al., 2003; Jitendra, 2013). In mathematics, students with LD are more likely to have difficulties planning and executing tasks, making connections between their prior knowledge and new mathematics content, and applying their knowledge to new situations (Archer & Hughes, 2011). Because explicit instruction focuses on teaching students how to complete the task through modeling and demonstrations, explicit instruction can minimize some of the challenges students with LD face when trying to problem-solve. Several studies have found that explicit instruction has been an effective method for supporting students with LD (Gersten et al., 2009; Graham & Harris, 2009; Hattie, 2009; Kroesbergen & Van Luit, 2003; Mastropieri et al., 1996; Swanson, 2001; Vaughn et al., 2000).

Even though explicit instruction has been proven as an effective teaching strategy for students with LD in mathematics, some researchers have argued that explicit instruction hinders students' development of a conceptual understanding of mathematics. Researchers have shown that students with LD made academic growth from pretest to posttest scores, and often this growth was statistically significant (Jitendra et al., 1998; Owen & Fuchs, 2002; Ross & Braden, 1991; Xin et al., 2005). During explicit instruction, teachers break down content in a way that gives students a series of steps to solve a given task. Through repeated exposure, students can solve the same type of tasks. When given a posttest, students show growth using this procedure. However, less is known about how accurate students are in identifying when they should apply this knowledge and if they can apply it to other mathematical contexts. For example, many studies on explicit instruction have a narrow focus on a mathematical skill such as solving one-step addition and subtraction problems (Gersten et al., 2009). Few studies have explored students' development of more complex algebraic concepts through explicit instruction. In their work, Hord and Newton (2014) explained

> In the short term, explicit instruction is potentially effective to help students solve problems more quickly; however, this earlier introduction of explicit instruction may slow the progress of students with LD in becoming resilient, persistent problem solvers and developing deep conceptual understanding of topics. (p. 198)

In her work, Lambert (2018) highlighted that researchers and educators sometimes assume that it might be too cognitively challenging for students with LD to construct their own knowledge. As a result, teachers rely on explicit instruction. However, Lambert argued that students with LD develop new mathematical knowledge based on their previous understanding, and, as such, they deserve to have access to standards-based instruction. Similarly, the National Mathematics Advisory Panel (Geary et al., 2008) noted that educators should not only rely on explicit instruction when educating students with LD in mathematics.

Student Voice

To design instruction that meets the needs of students with LD in mathematics, there is a need to know which instructional approaches they believe work best for them. Often, research on students is from an objectivist paradigm in which researchers study students as objects that can be observed and measured (Gentilucci, 2004). However, Gentilucci (2004) noted that researching students from an objectivist paradigm fails to take into consideration the thoughts and feelings of students about their learning. Without considering students' voices, it can be difficult to change instruction in a way that supports the improvement of student outcomes (Cook-Sather, 2002; Gentilucci, 2004; Mitra, 2003). For example, Cook-Sather (2002) explained that if the voices of students are excluded from research, then the full picture of education in both the classroom and school-wide is incomplete. As a result, educators cannot address the needs of students if those needs are unknown. To highlight the importance of student voice in educational research, Hammersley and Woods (1984) stated

> There can be little doubt that pupils' own interpretations of school processes represent a crucial link in the educational chain. Unless we understand how pupils respond to different forms of pedagogy and school organizations and why they respond in the ways that they do, our efforts to increase the effectiveness, or to change the impact of schooling will stand little chance of success. (p. 3)

Sometimes teachers and mathematics education researchers make assumptions about the ways in which students learn best. However, students have knowledge, perspectives, and opinions that are different from adults and are based on their own unique experiences (DeFur & Korinek, 2010; Mitra, 2003). More often than not, adults cannot replicate the perspectives of students (Mitra, 2003). The voices of students should be included in research to serve students throughout their education. Furthermore, it was important to frame this study around the inclusion of student voice because teachers without a disability label have never experienced instructional practices in the same way as students with LD.

The belief that giving voice to students with the notion that using their understanding can improve mathematics instruction guided the design of this study. Often schools and classroom instruction are not structured in a way that gives voice to students. As a result, schools do not adequately address students' needs, and in some cases, their needs conflict with the structure of schools (Costello et al., 2000). However, in her work, Mitra (2003) stated

> Consulting with students on their views of teaching and learning has improved students' understanding of how they learn, helped students gain a stronger sense of their own abilities, and improved instruction so that teachers do a better job meeting student needs. (p. 3)

Because students are the ones receiving the instruction, they are experts in the classroom experience (DeFur & Korinek, 2010; Mitra, 2003). Students "have singular and invaluable views on education from which both adults and students themselves can benefit" (Cook-Sather, 2002). Furthermore, researchers and educators should attempt to link student voice with curriculum and instruction in an effort to improve instructional practices, student engagement, and student outcomes (DeFur & Korinek, 2010; Oldfather, 1995; Rudduck & Flutter, 2000). In her work, Mitra (2003) studied general education students providing feedback to their teachers. Students shared the types of classroom and teaching styles that worked best for them. Furthermore, teachers valued this feedback and considered it when planning future lessons. Some teachers began to collaborate with students on a regular basis to modify the curriculum and receive feedback on instructional strategies and pedagogy. Teachers felt that incorporating student voice into their work was advantageous. In addition, listening to students can make instruction more accessible (Commeyras, 1995; Dahl, 1995; Johnston & Nicholls, 1995). Because students with LD within an ICT setting were the focus of this study, the notion of accessibility was important. In the work of DeFur and Korinek (2010), students with disabilities shared that teachers should know their students' needs and behaviors, and more importantly, they should know how to address those behaviors. Additionally, Cook-Sather (2002) suggested that researchers and educators ask students directly about their preferences for instruction and repeatedly ask because answers are not universal and will differ among contexts and students.

In terms of the field of mathematics education, Lambert and Tan (2017) argued that students with LD and their voices are excluded from research because they are framed as "problematic." In their work, Lambert and Tan (2017) wanted to explore the ways in which mathematics research addressed and studied students with disabilities and students without disabilities to identify a potential divide in research. Lambert and Tan identified 149 peer-reviewed articles published between 2013 and 2015 that focused on students' mathematical problem-solving in kindergarten through 12th grade. They paid particular attention to the theoretical framework and methods used within these studies. Lambert and Tan found that 86% of research studies on students with disabilities were quantitative in nature compared to 35% of research studies on students without disabilities. While 50% of research on students without disabilities were qualitative, only 6% of studies on students with disabilities were qualitative. More importantly, Lambert and Tan highlighted that students with disabilities were often studied through aggregate test scores. As a result, there was little to no analysis of individual student thinking. By studying students primarily through quantitative methods, Lambert and Tan (2017) stated that

> Students with mathematical learning disabilities for example, often have particularly unique ways of approaching mathematics, yet research on these students is typically not sensitive to individual differences, instead seeking to understand all individuals with a single disability as a unified group. (p. 14)

Similarly, researchers attempted to understand how students with disabilities solve mathematics problems through the medical, behavioral, and information processing approaches. However, for students without disabilities, their mathematics work was understood through the constructivist and sociocultural approaches (Lambert & Tan, 2017). Both the constructivist and sociocultural theories were used to frame this study on students with LD.

Very few studies that focus on the perceptions of students with LD on their mathematics instruction exist. In their work, Leafstedt et al. (2007) interviewed high school students with LD regarding their experiences in an ICT classroom. While this study did not focus on mathematics instruction, Leafstedt et al. (2007) noted that students "were acutely aware of how they learned and how they wished to be taught" (p. 180). Not only did students explain that there were differences in the ways in which the general education teacher and the special education teacher presented materials, but they also shared that the special education teacher was better equipped to meet their needs. Additionally, students identified their learning preferences, which included teaching content slowly, breaking content down into steps, and explaining the content in different ways. Students highlighted that they benefitted from teachers differentiating and individualizing instruction, but that this type of practice happened less frequently within the general education classroom. Even when students with LD received their instruction from the special education teacher in the general education classroom, students still felt that their needs were not met. The purpose of the current study was to gain insight similar to the work of

Leafstedt et al. (2007), in which students with LD articulated the types of instruction that did and did not work for them. However, this study paid particular attention to mathematics instruction within an ICT setting. To support the academic development of students with LD, the design of this study was based on understanding the ways that students with LD believe that they learn best, which includes their perceptions of their instruction and their educational setting in mathematics.

Structure of the Dissertation

This dissertation was written in a manuscript style in which Chapters II to IV are stand-alone articles that will be submitted to different peer-reviewed journals. Chapter II addresses the first research question, "Within an ICT setting, what types of instructional practices do students with LD perceive as supportive for their success in mathematics?" Chapter II presents the findings from an interview study of high school students with LD who were enrolled in an ICT mathematics class. In this study, the researcher conducted semi-structured interviews with the purpose of honoring the voices of students with LD. Interview questions aimed to understand how students with LD perceived their instruction in mathematics with a particular interest in practices that they found beneficial and supportive of their learning and those they felt were disadvantageous. Because research on high school students with LD in mathematics is limited (Beatty & Bruce, 2012; Geary et al., 2008; Gersten et al., 2009; Lambert & Sugita, 2016; Lambert & Tan, 2016; Lambert & Tan, 2017; Watt et al., 2016), the researcher sought to understand the ways in which students with LD wanted to be taught in an effort to support them in the classroom better. Overwhelmingly, participants preferred explicit instruction, which involved teachers breaking down content and incorporating guided practice and repeated exposure to mathematics concepts. Participants highlighted that the pace of their mathematics instruction was usually too fast, and they often found it difficult to keep up. Additionally, participants shared ways that their teachers ensured their understanding of the mathematics content. This article will be submitted to *Theory, Research, and Action in Urban Education*.

Chapter III addresses the second and third research questions, “What conceptions of linear functions do students with LD possess, as evident in their work on problems with abstract graphical representations and real-world connections?” and “Based on existing literature, to what extent, if any, does the way in which students with LD approach tasks on linear functions differ from students without disability labels?” For this part of the study, the researcher utilized mathematical task interviews, also known as clinical interviews (Clement, 2000), to develop an understanding of the content knowledge of linear functions of students with LD. After participants completed the semi-structured interview that was included in Chapter II, the researcher conducted a three-part mathematical task interview with the same participants. Because students with LD often perform lower in mathematics on standardized tests such as the NAEP, in particular on the algebra subtest (Cortiella & Horowitz, 2014; Watt et al., 2016), there is a need to know their current level of understanding in order for teachers to provide them with appropriate, rigorous, and meaningful instruction. Additionally, the researcher considered the ways that participants’ thinking aligned or differed from the existing literature and research studies on linear functions for students without disability labels. On both mathematical tasks that were included within this study, participants demonstrated a procedural understanding of linear functions. While five of the six participants recalled appropriate procedures, their accuracy in applying these procedures varied. When asked to explain the meaning of the rate of change and y-intercept, participants showed a limited conceptual understanding of linear functions as they struggled to describe the rate of change and y-intercept in relation to the context of a real-world problem. This article will be submitted to *Learning Disabilities Quarterly*.

Chapter IV is a resource to support teachers, both general education and special education, in their effort to encourage the development of a conceptual understanding of linear functions for students with LD. While based on existing literature of students without disability labels, this article attempts to highlight the importance of developing a conceptual understanding, and it provides suggestions for teachers on the ways that they can implement instruction within their classroom. The article begins by explaining students' content knowledge of the rate of change. In particular, the article highlights students' reliance on procedural knowledge when finding the rate of change. Next, the article addresses the potential benefits that transpire when teachers design instruction that incorporates a conceptual understanding. One way in which teachers can encourage a more in-depth understanding is through strategically incorporating real-world problems into their instruction. However, it is noted that teachers should use caution when integrating real-world problems. Students can make incorrect connections using their prior knowledge, which leads them to develop misconceptions. Within the article, a table of common student misconceptions is included, and an appropriate research-based intervention is suggested to address each misconception or misunderstanding. Finally, the article discusses strategies that teachers can utilize to incorporate research-based practices for students with LD, such as integrating explicit instruction, heuristics, and student verbalizations. This article will be submitted as a research brief to the National Council of Teachers of Mathematics.

Chapter V suggests that results from Chapter II and Chapter III can be used to begin to bridge two different divides that exist within the literature and the classroom. The hope is that by bridging these gaps, teachers and mathematics education researchers can better serve students with LD in mathematics. The first is the divide between the amount of qualitative research conducted on students without disability labels and those with disability labels. Chapter V addresses the importance of utilizing qualitative methods, such as interviews, to give voice to students with LD. Results from Chapter II and Chapter III, which utilized interviews, begin to fill this problematic gap in the literature, and show the value of giving students with LD the opportunity to share their knowledge and experience. The second divide is the one that exists between explicit instruction and inquiry-based pedagogy. Even though participants in this study shared that they preferred explicit instruction, they also explained that they wanted their teachers to provide them with multiple problem-solving strategies. Furthermore, participants wanted their teachers to allow them to select whichever problem-solving method they felt most comfortable using. The idea of allowing students to use strategies based on their knowledge and strengths aligns with inquiry-based instruction. As such, Chapter V provides suggestions for mathematics education researchers and educators on how they can integrate instances of various types of instruction to best meet the needs of students with LD within an ICT class.

CHAPTER II

"SOME KIDS UNDERSTAND THINGS DIFFERENTLY:" PERCEPTIONS OF STUDENTS WITH LD ON THEIR MATHEMATICS INSTRUCTION IN AN INTEGRATED CO-TAUGHT SETTING

Abstract

This study investigated the perceptions of high school students with a learning disability (LD) educated within an Integrated Co-Taught (ICT) setting for mathematics. Semi-structured interviews were conducted with six students with LD. Guiding questions were used to explore instructional practices that students with LD found advantageous and disadvantageous. Results indicated that students preferred to receive explicit instruction in which their co-teachers broke down mathematical content into steps and incorporated guided practice and repeated exposure. Participants also valued when their co-teachers gave them the opportunity to ask questions and participate in group work. However, participants felt that their co-teachers did not always meet their learning needs, particularly regarding the speed of their instruction and how they explained the content. The results are discussed in terms of the following four major themes: (a) breaking down content, (b) pacing, (c) ensuring student understanding, and (d) group work.

As the number of students with a learning disability (LD) educated within a general education classroom increases, the need to identify instructional practices that best support students with LD in mathematics is imperative. The Individuals with Disabilities Education Act (IDEA; 2004) mandates schools to provide students with LD access to the same standards-based curriculum as students without disability labels (Cramer, 2015; Jitendra, 2013; Lambert, 2018). Although the intent of educating students with LD within a general education classroom is to improve academic performance, many students with LD struggle academically compared to their peers without disabilities in mathematics (Cortiella & Horowitz, 2014). In addition to earning lower grades on standardized tests, such as the National Assessment of Educational Progress (NAEP), and on in-class assessments and assignments, as a group, students with LD are more likely to experience higher rates of course failure compared to students without disabilities (Cortiella & Horowitz, 2014). Furthermore, in the United States, compared to approximately 85% of the total population of students (National Center for Education Statistics, 2019), only 68% of students with LD receive a regular high school diploma (Cortiella & Horowitz, 2014).

One common way schools in the United States educate students with LD within a general education setting is through Integrated Co-Teaching (ICT; Scruggs et al., 2007). Within an ICT class, a content teacher and a special education teacher work together as co-teachers to co-plan, co-instruct, and co-manage a group of students with varying knowledge and skills (Murawski & Lochner, 2011). Co-teachers must present instruction that meets the learning needs of a diverse group of students and improves the performance of all students within the class (Cook & Friend, 1995; Cook et al., 2017; Friend, 2008; Mastropieri et al., 2005). The premise is that an ICT setting gives "students with disabilities access to the general education curriculum, but also provides the specialized instruction they need to succeed" (Friend, 2016, p. 17). Explicit instruction is an effective strategy for teaching students with LD. However, to encourage students' development of a conceptual understanding of mathematics, co-teachers should consider and incorporate other practices that aim to promote classroom discussion and discourse (Stein et al., 2015). Much of the literature on teaching practices that foster a conceptual understanding is based on students without disability labels, as research is limited on students with LD in mathematics.

Teaching Practices to Support Students with LD in Mathematics

Explicit instruction is a commonly used practice to facilitate high-quality instruction within a whole class (Doabler & Fien, 2013; Hughes et al., 2017) or a small group setting (Archer & Hughes, 2011). During explicit instruction, the teacher models a new mathematical concept by breaking it down into discrete steps in an effort to reduce the cognitive load placed on students (Archer & Hughes, 2011; Doabler et al., 2012; Weibe Berry & Namsook, 2008). Once the demonstration is complete, the teacher leads students in guided practice. During guided practice, the teacher provides more examples of the same mathematical concept, and students begin to take on some of the responsibility of solving the task (Doabler & Fien, 2013). While asking questions and eliciting student participation, the teacher monitors students' responses and provides appropriate and timely feedback (Doabler & Fien, 2013). After guided practice, the teacher invites students to work independently, in pairs, or in small groups, to complete a similar task or series of tasks.

In addition to being named a "High Leverage Practice" by the Collaboration for Effective Educator Development, Accountability, and Reform (CEEDAR) Center (McLeskey et al., 2017), researchers have shown that students with LD benefit from explicit instruction (Bryant et al., 2003; Jitendra, 2013). After analyzing the results of 11 studies using targeted interventions for students with LD in mathematics, Gersten et al. (2009) found that explicit instruction had a large and meaningful effect on students' mathematics achievement, often measured by an increase in scores from pretest to posttest. Similarly, in his review of over 800 meta-analyses, Hattie (2009) found that explicit instruction had a medium to high effect size on student achievement. Several other studies have found that explicit instruction has been one of the most effective methods used to teach students with LD (Graham & Harris, 2009; Kroesbergen & Van Luit, 2003; Mastropieri et al., 1996; Swanson, 2001; Vaughn et al., 2000).

Researchers have found that students with LD as a group made academic growth from pretest to posttest scores and that growth was often statistically significant (Jitendra et al., 1998; Owen & Fuchs, 2002; Ross & Braden, 1991; Xin et al., 2005). However, less is known about how accurate students are in identifying when and how to apply this knowledge to other mathematical contexts. For example, Gersten et al. (2009) explained that many studies on explicit instruction had a narrow focus, such as finding the value of half of a quantity or solving one-step addition and subtraction problems. Few studies have explored students' development of more complex algebraic concepts through explicit instruction. In her work, Lambert (2018) highlighted that researchers and educators sometimes assume that it might be too cognitively challenging for students with LD to construct their own knowledge. As a result, teachers rely on explicit instruction. However, both Lambert (2018) and the National Mathematics Advisory Panel (Geary et al., 2008) argued that explicit instruction should not be the only method used to teach students with LD in mathematics.

In addition to explicit instruction, researchers have studied other teaching practices and pedagogy in an attempt to determine a relationship between practices and improved academic performance of students with LD. In their review of 42 quantitative studies over a 30-year period, Gersten et al. (2009) found that student verbalizations of mathematical reasoning, range and sequence of examples, teacher feedback, and student feedback were beneficial for students with LD. Similarly, Watt et al. (2016) conducted an analysis of research on students with LD in algebra. While they found that the concrete-to-representational-to-abstract (CRA) approach was the most common intervention, other studies utilized peer tutoring, heuristic or mnemonic devices, and graphic organizers.

Encouraging Student Participation and Discussion

Participating in productive mathematical discourse may help students develop their conceptual understanding (Stein et al., 2015). Teachers play an essential role in facilitating whole-class discussion. One way teachers can do this is by asking open-ended questions. Posing an open-ended question provides all students, even students whose skills are still emerging and students with disabilities, access to some part of the task (Hoffer, 2016; Kendrick, 2010; Piccolo et al., 2008). Furthermore, open-ended questions give students a chance to solve the task in any way that they prefer. In their research, Manouchehri and Enderson (1999) found that by designing and implementing open-ended tasks to allow students to engage in authentic math inquiry, classroom discussion was robust. However, when teachers administered close-ended tasks, students completed the task independently and simply checked their final answers with their classmates. If students agreed on their final answers, little discussion occurred about the process taken to answer the question or students' mathematical thinking (Lack et al., 2014).

Teachers can limit student discourse and student participation through their actions in the classroom. Students usually direct their questions and answers to their teacher. For example, in an eighth grade mathematics class, students directed 88% of their comments toward their teacher (Mendez et al., 2007). In addition, Shepherd (2012) explored a teacher's checks for understanding during a teacher-led discussion and found that the teacher did not acknowledge 74% of students' attempts to participate. Even though teachers may not intend to limit student participation in discussion, their actions and practices result in fewer opportunities for students to participate in authentic mathematics discourse.

Effectively incorporating and facilitating group work can help teachers encourage student-to-student discussion. When teachers monitored student group work and interacted with students, Gillies (2004) found that students in grades five through seven gave more detailed explanations of their understanding than students whose teachers provided explicit instruction. Additionally, Kazemi and Stipek (2001) highlighted that elementary school students with teachers who probed them during group work were more likely to provide detailed explanations and justify their problem-solving strategies than students with teachers who did not probe students. Instead, these students only summarized or listed the steps they took to solve the problem. As a result, little mathematical discussion occurred. However, DeSimone and Parmar (2006) shared that students with disabilities in middle school did not actively participate when teachers incorporated group work.

Perceptions of students with LD of Instruction

As students within an ICT class are consumers of instruction, research should privilege their lived experiences (Lambert, 2016). While limited, research that has included the voices of students with disabilities within an ICT class has focused primarily on the ways in which students perceive their co-teachers and the co-teaching models utilized during instruction. For instance, King-Sears and Strogilos (2018) found that students and co-teachers perceived that the *one teach, one assist* co-teaching model was used most frequently, and they found that students viewed the general education teacher as the lead instructor. Furthermore, there is limited research about how students with LD perceive the way in which content is presented within an ICT setting, and even less is known in terms of mathematics instruction (Strogilos & King-Sears, 2019). While not focusing particularly on mathematics instruction, in their study of high school students with LD, Leafstedt et al. (2007) found that students preferred to receive their instruction in a resource room setting in which they were separated from the ICT class. These findings may possibly be due to the pace of the lesson, the style of teaching, or the number of students in the class. Leafstedt et al. (2007) noted that

> Students gave a great deal of importance to the special education teacher being able to teach fewer students, change the pace of the lesson, and teach in a different manner within the special education setting. This is in conflict with the rationale for co-teaching, which states that students will receive a wider range of instructional options when in a co-taught classroom. (p. 182)

Not only did students with LD within this study share that they felt their instruction within an ICT setting was ineffective or overwhelming, but they also articulated the support they needed from their teachers and the ways that their teachers could meet their needs.

In theory, co-teachers have a more comprehensive range of teaching strategies that they can implement in an effort to meet the needs of a diverse group of students. However, while proven as an effective teaching practice for students with LD, the use of explicit instruction conflicts with the priority given to the active construction of knowledge through exploration and inquiry that is encouraged by the National Council of Teachers of Mathematics (NCTM). This clash in instructional pedagogy may create a potential challenge for co-teachers as well as students with LD. Very few studies have explored how students with LD perceive these instructional practices and the ways that their teachers implement and encourage participation in a standards-based curriculum. For example, Lambert and Sugita (2016) found only seven qualitative peer-reviewed studies that showed evidence of the inclusion of students with disabilities in classrooms implementing a standards-based curriculum. None of these studies included high school students (Lambert & Sugita, 2016). Furthermore, in a review of 15 studies of students with LD in algebra, Watt et al. (2016) found that only 44% of the participants were Black, and only 13% were Hispanic.[1] As such, there is a need to gain a more diverse perspective of students with LD.

As most of the research on students with LD in mathematics is quantitative in nature (Gersten et al., 2009; Lambert & Tan, 2017; Watt et al., 2016), the perceptions of students with LD about their mathematics education are missing from the literature (Lambert & Tan, 2016). Due to the limited research on the participation of students with LD in standards-based mathematics classes, it is unknown which instructional practices their co-teachers are using, which they prefer, and which they believe are advantageous to their learning. To support students with LD within ICT mathematics classes, there is a need to know more about the ways in which they perceive their mathematics instruction. Thus, the following overarching research question and sub-research questions were developed:

Research Question: Within an ICT setting, what types of instructional practices do students with LD perceive as supportive for their success in mathematics?

Sub-Research Question 1: How do students with LD in an ICT setting prefer to receive instruction in mathematics?

[1] Cited as Hispanic in Watt et al.'s (2016) study

Sub-Research Question 2: What, if any, instructional practices do students with LD believe as disadvantageous to their learning experience in mathematics?

Method

To address the research questions, an interview study was conducted. Semi-structured interviews were conducted with Black and Latinx high school students with LD.

Participants

Six students with LD who attended a public high school in a large urban school district in the northeast United States at the time of the study participated in this research. Participants attended different schools throughout this urban area. All participants met the following criteria: (a) had an Individualized Education Program (IEP); (b) self-identified as having a learning disability; (c) were enrolled in ninth, 10th, 11th, or 12th grade at an urban high school; (d) were enrolled in an ICT class for mathematics; and (e) gave student assent and obtained parent or guardian permission to participate. Table 1 shows the demographic characteristics of each participant at the time of the interview.

Table 1

Participant Demographics at Time of Interview

Pseudonym	Sex	Age	Grade	Race/Ethnicity
Julie	Female	17	12	Black
Joshua	Male	17	12	Latinx
Orlando	Male	17	11	Black
Michael	Male	16	11	Latinx
Felix	Male	16	11	Black
Maura	Female	15	10	Black

Note. Demographics self-identified by participants.

At the time of the study, none of the participants were classified as English Language Learners, nor were they receiving special education services for speech. The six participants in this study attended schools in which all students received free or reduced lunch. Additionally, it should be noted that the researcher did not gather the specific classification of LD for each participant. Rather than only including students with mathematics disabilities, all students with LD were included because the urban area in which this study was conducted does not differentiate between the types of LD on students' IEPs. Unless students go to an outside agency for a diagnosis, the specific type of LD may be unknown to students, parents, and teachers.

The researcher recruited participants by posting flyers at local community centers and snowball sampling. Six participants were included, and saturation across cases and amongst individual participants was evident. In terms of data across cases, the same codes were used, and no new codes or themes appeared when analyzing data (Urquhart, 2013). Similarly, data saturation was reached amongst individual participants, as enough data were collected from each participant to understand their specific point of view (Legard et al., 2003).

Data Collection

This study was part of a larger study in which participants completed a two-part interview with the researcher. The first part of the interview was a semi-structured interview, which explored participants' perceptions about their mathematics instruction within an ICT setting. During the second part of the interview, participants completed a series of mathematical tasks with the purpose of sharing their understanding of linear functions. Only data from the semi-structured interviews were included in this study. Semi-structured interviews were conducted over a two-month period between July and August. All semi-structured interviews were conducted by the researcher and lasted approximately 20 to 25 minutes. Due to the pandemic caused by COVID-19, the researcher conducted all semi-structured interviews virtually through Zoom. The interviews were recorded and subsequently transcribed.

With the purpose of giving voice to students with LD, the researcher asked participants to describe their experiences within an ICT mathematics class in high school. Interviews were semi-structured to allow the researcher to explore certain subjects in greater depth (Patton, 2002). As such, interview questions were broad, including questions such as, "Describe some of the things that your mathematics teachers do that you find most helpful when learning math concepts" and "Thinking back to all of your math classes and co-teachers in the past, which do you think were the best at teaching you math concepts and why?" Semi-structured interviews also included several follow-up prompts such as, "What are some things that you like about your math instruction?" and "What are some things that you dislike or that you would like to change?" Interview questions were piloted with a focus group of eight middle school students with LD prior to the start of this study. The researcher did not identify any issues in the structure of the interview or the content. Based on the feedback from the pilot focus group, no changes were made to the sequencing or wording of the questions.

Data Analysis

Analysis of data was informed by the research questions (Creswell, 2015). The researcher began by analyzing data holistically across all six participants for significant instances related to participants' perceptions and beliefs about common and beneficial instructional practices within their ICT mathematics classes. Data analysis began with in vivo coding to ensure that codes were participant inspired rather than researcher generated (Saldaña, 2009). Based on key terms and phrases that continued to appear throughout and across transcripts, 16 in vivo codes were identified. Using these 16 in vivo codes, the researcher coded line by line of each transcript.

After each line was coded, the researcher organized the 16 in vivo codes and data units into major and minor themes, uncovering both complementary and contradictory themes (Patton, 2015). Originally, the researcher uncovered five major themes. After review, the researcher determined that some in vivo codes overlapped. As such, the researcher collapsed two major themes, and found four major themes across cases. Finally, the researcher utilized a member checking session with all six participants regarding the researcher's categorical assertions and conclusions (Patton, 2015). Participants agreed with the researcher's conclusions, and they believed that these conclusions aligned with their preferences and experiences.

Results

Four themes emerged from the interviews with students with LD about their mathematics instruction in an ICT setting: (a) breaking down content, (b) pacing, (c) ensuring student understanding, and (d) group work. Each theme is described in detail.

Breaking Down Content

To some extent, all six participants mentioned that they wanted their mathematics teachers to "break down" new mathematics content. Not only did they each mention the term "break down," but participants also described their meaning of "breaking down" content in a similar manner. To the participants, breaking down the content meant that teachers separated a problem into a series of smaller pieces or steps. Students followed along as teachers showed them how to solve the problem step-by-step. Participants shared that learning new content through steps helped them to understand the problem better, solve the problem, and be able to utilize their work as a reference point for future problems. The following quotes are indicative of participants' perspectives about their preference for co-teachers to break down mathematical concepts into steps.

I feel like step-by-step in everything from the beginning to the end is best. I want them to go step by step on where I have to add, where I have to multiply, so that I can put it down on my paper, and when I do the next question, I can remember it. I can use the other question and try to solve the next question the same way and try to remember it. (Michael)

I have a problem with comprehending questions, so the teachers will break down the question for me. They will help me with the first few steps of the question and then show me what to do afterwards, and then tell me if I am doing it wrong or if I am doing it correct. I thought this was very helpful. (Felix)

It was helpful that we had her break down the problem into steps for us to understand. (Maura)

I think it is helpful for problems with a lot of steps to have some directions or a list so that I know what to do for each step. (Julie)

Joshua, who was entering his senior year of high school at the time of the study, mentioned that he favored his co-teachers during his sophomore year the most because his teachers knew when they could push him and his understanding further. One way they did this was by having him explain the work to his classmates. Joshua noted that this practice helped him to develop his understanding better as he learned to break down the topic in his own way.

I think explaining it helped me to understand the work even more because I had to explain it. I had to break it down for the other students and that like helped me to learn how to break it down and understand the steps. (Joshua)

Several participants shared that their teachers used visual reference points to help break down content for them, such as anchor charts or notebook pages. Participants explained that their teachers gave them a resource that not only showed each step of the problem, but also that they could "look back at" when they were stuck on a similar task.

> *Umm ... they used posters to help us remember certain parts of the lesson. That was very helpful. I could ... I forgot a lot of stuff, and so with the posters, I was able to look at the posters and follow the steps. I would look at the poster to try to figure out the problem that I was having problems with.* (Felix)
>
> *Maybe like posters in the room with some of the stuff that we have learned. Give me examples so I can look at the example and if I get stuck, I could look back.* (Julie)

While some participants mentioned posters hanging around the room or on the board, others discussed that using their notebook was also advantageous for them. For instance, Felix used his whole notebook as a tool to develop his understanding by looking for similar concepts and problems. However, Maura explained that her teacher encouraged her to use sticky notes on key pages that would be beneficial for her to reference quickly and easily.

In addition to breaking down mathematics content into discrete steps, participants emphasized that they preferred when their co-teachers included guided practice during instruction. Even though teachers can give students steps to solve a problem, participants felt it was imperative for their learning that they had a chance to practice their new knowledge under the guidance of their co-teachers because it gave them the opportunity to ensure that they understood the content before completing their work independently or in small groups.

> *I like how she would do one problem together, and then we would do one problem on our own, and she would check it before we would go into groups or do our worksheet.* (Maura)

It was helpful that I could do the work on my own and then check it with the class before we had to do it all on our own. (Orlando)

They would break down each question to show us the steps to solve and let us try it so we could practice it. For me, when it comes to math, I have to be able to see my teachers do the problem on the board and I have to be able to write the problem out at the same time. To do it along with the teacher. (Felix)

Participants favored repeated exposure during explicit instruction and guided practice when learning a new mathematical concept or skill. Not only did they want a chance to practice with their teacher as a whole class, but participants also wanted the opportunity to review and practice the series of steps multiple times. When asked about their mathematics instruction, participants wished that their teachers gave them more opportunities to apply their knowledge during the guided practice portion of the lesson. Because of limited repeated exposure, participants did not feel confident with their understanding of the new mathematical concept before working independently.

I think the teacher has one type of problem and they break that problem down. But not just one time. They need to do it a few times and let us try it a few times. (Joshua)

We really only do two questions together before we go on our own. So, I wish she would do maybe another one or two questions so that we really understand it and we can practice it before going on our own. I have to like … I have to try it on my own before I fully get it. (Maura)

But, you know, we really didn't get that much. I think they would just show us the problem only a few times before we had to do it on our own. (Julie)

All participants commonly mentioned references to explicit instruction, guided practice, and repeated exposure. Additionally, all participants shared that new content was presented through explicit instruction rather than through exploration or inquiry tasks. Although participants shared that they favored explicit instruction, they wished their teachers gave them more opportunities to work through problems together to ensure they understood the work before they were asked to complete their work independently or in groups.

Pacing

The pace of explicit instruction was a popular topic amongst all participants. Five out of the six participants felt that their mathematics instruction was rushed. Participants described instances in which their co-teachers would move on to the next problem or topic before they fully understood or had a chance to ask questions and receive clarification. In terms of daily instruction, participants described instances in which their co-teachers moved on to the next question or task before they felt ready.

> *Sometimes she will move on to the next problem without like fully explaining it so that we understand it. She just moves on and some of us still have questions. I wish she would review it more.* (Maura)
>
> *They didn't give us enough time to finish our work.* (Felix)
>
> *He doesn't let the kids do it on their own for like five minutes instead of like …, 'Oh you don't understand? Okay, I am going to do it on the board all together.'* (Julie)

The pacing of daily instruction was not the only concern of participants. Participants described that their co-teachers moved from one topic to another too quickly as well.

> *Sometimes because we would only spend two days on it, it went a little too fast for some of us.* (Michael)

> *Well, for me, I feel like it is a little rushed to be honest. Everything we do, when they teach you. Just when I thought I would be getting it, they would move on to the next thing.* (Orlando)
>
> *She goes over things quickly. She teaches some topics once and then she moves on and we do not always understand before she moves on. I think some kids might think it is hard because of the pace of the class*. (Maura)
>
> *Some of them went too quickly and definitely did not go over the work enough. They would go over it like maybe two or three days and they would think that everyone understands so they would move on, but not everyone did understand. And then kids might be asking for help but they just move on.* (Joshua)

While a majority of the comments about instructional pacing highlighted that the speed was too fast for participants, two participants shared instances in which their favorite co-teachers gave them a chance to complete their work at a pace that was sufficient for them.

> *So my teachers never really went too fast for me because some of the other students couldn't keep up. They sort of made sure that everyone had the notes before they would move on. I think that is important. If the pace is too fast, then I cannot follow along.* (Felix)
>
> *I think that he really showed us what we needed to learn as a class and then he gave us the time to practice it.* (Joshua)

With that said, Felix and Joshua noted that this was not common throughout their mathematics instruction in high school.

Ensuring Student Understanding

Throughout the interviews, participants discussed several ways in which either they or their co-teachers monitored and ensured participants' understanding of the mathematics content. Participants shared that they could better understand the work when their teachers presented and explained the content in different ways. Several participants mentioned that all students learn differently, and as such, teachers may want to consider those differences when teaching and re-teaching mathematical concepts. Not only did participants highlight that they wanted the content presented in different ways, but they also wanted the opportunity to select and utilize the method that worked best for them.

> *I like when they do not limit us to one method or one way to solve something. They showed us other ways. If this way is not working for me or it is difficult for me, they showed me different ways. I had the chance to pick which way worked for me. Everyone could pick which worked for them because it is different for me than it is for someone else in the class. I know that not everyone learns in the same way and likes the same method.* (Joshua)

> *I feel like sometimes her saying stuff in different ways is good. Some kids understand things differently. So, you could tell me something in a certain way and then explain it in another way with different vocabulary and some kids would understand it that way.* (Michael)

> *I like when teachers explain it in so many different ways that all of us would understand it in our own unique way.* (Orlando)

Similar to preferring that their teachers explain the mathematics content in multiple ways when asked what they disliked about their mathematics instruction, participants indicated when their teachers limited them to only one method of solving a problem.

Another teaching practice that participants discussed was the willingness of their co-teachers to "go over it again." When asked about reviewing content in which they found difficult or did not understand, participants described their co-teachers as willing to work with them one-on-one, in a small group, or as a whole class to review the problem or content again. Furthermore, participants shared that they found this helpful and appreciated it when teachers would take the time to re-explain the problem to them.

> *One of my teachers, she would go around when we were doing our work by ourselves and check and see if we understood it properly and if we didn't, she would go over it again or ask another student to help us. If everybody needed it explained again, they would just explain it to the whole class again.* (Felix)
>
> *She will come to us when we do not understand and explain it again.* (Maura)
>
> *The teachers will go over it again, and if I need help with the problem, they would give me hints. They won't do the work for me. They would ask me questions about what I would do next. They would read the directions and then they would be like, 'What do you think you are supposed to do?'* (Julie)

Even though participants highlighted instances in which their co-teachers answered questions and re-explained content to ensure student understanding, some participants felt that their co-teachers did not always explain the work in a way that made sense to them. Additionally, a few participants mentioned that their teachers would re-explain the content in the same way they initially taught it. While they were appreciative that they could ask their teachers to re-explain the concept, participants said that explaining it and re-explaining it the same way was not helpful for their understanding.

They need to be more flexible with how kids learn and understand and behave. They would only teach it one way. They like almost didn't care if we didn't get it. They just expected us to understand it in the way that they taught it. And if we didn't, they were going to move on anyway. (Joshua)

The way they explain it is like they are explaining it to people their age. They are not trying to explain it to us. They only explain it one way, and we are expected to just get it the way they explain it. If we don't understand it, they will help, but usually they just explained it again the same way. That is not helpful. (Orlando)

Other participants shared that even when teachers did re-explain the content, they still struggled to develop an understanding of the concept. As a result, participants simply followed along with the lesson. Because they felt that their teachers did not explain the content in a way that made sense to them, Michael and Orlando mentioned that they took the initiative to teach themselves.

Sometimes I wouldn't understand some of the stuff that they were saying, but I would like try to teach myself. (Michael)

I would watch YouTube videos and try to understand it in my own unique way. But it was a challenge that I needed to have to let me know that not every teacher is going to meet my needs. There were so many students that I had to fend for myself. (Orlando)

Four of the six participants highlighted that they could ask their teachers questions or ask for help if they did not understand their mathematics work. For the most part, participants felt that being able to ask their co-teachers questions was helpful for their learning.

If I still didn't get it, I would ask the teachers questions or ask for help. (Julie)

When we have to ask questions, they are always willing to listen. (Michael)

It was helpful for him to come together at the end to go over it so that we knew immediately whether our work was correct or incorrect and then we could ask him questions before we did the exit ticket. (Joshua)

However, two participants shared that some of their mathematics teachers limited either their or their classmates' attempts to ask questions during class instruction.

She just moves on and some of us still have questions. (Maura)

Sometimes because she knew I understood everything that she was teaching, she wouldn't really focus on letting me ask her questions. She would let other kids ask questions. But I was asking questions for the other kids. Like I was trying to help the other kids because not all of them wanted to raise their hands. They may have been too scared and they didn't want to be judged. (Joshua)

During the interview, participants were not asked directly about seeking extra help. However, five of the six participants described instances in which they received help from their teachers outside of their mathematics class. Not only did participants state that they sought out their teachers during lunch or after school, some even took it upon themselves to receive extra help from other teachers in their school.

Even if I didn't understand it, they would always try to keep me after school or go to programs or come up at lunch. They were always there to help us if we needed it. (Michael)

She tells us to come after school and we will go over it together. She will pull us at lunch and make sure we fully understand everything. (Maura)

They were very helpful and every time you needed help, they would always try to make the time. I would literally beg her to do Saturday school so she could help me. (Julie)

Me, personally, I would go to another teacher. I would go to her for help and she would explain it to me. She was a special education teacher. I really like her style and how she taught. And then I would take her style and apply it to class. (Joshua)

During their interview, participants shared several different ways in which their teachers sought to ensure student understanding. Often participants had the opportunity to ask their teachers questions or to review and re-explain a problem. However, participants noted that their teachers did not always explain the work in a way that allowed participants to access the content. Additionally, many participants had to take ownership of their learning and get extra help during their free time, such as at lunch or after school. Other students found it essential to teach themselves. Overall, there was a wide range of participants' responses regarding how their teachers ensured that they understood the mathematics content during classroom instruction.

Group Work

Some participants described instances in which their teachers included partner and group work into their daily instruction. After co-teachers finished their explicit instruction and guided practice, participants explained that their co-teachers put them into groups. Participants preferred group work as an opportunity for them to work with and learn from their classmates. In addition to being given the opportunity to explain their knowledge to their classmates, participants shared that they could develop a deeper understanding of the content by listening to the perspectives of their peers.

She made us do group work, and then we could help each other in our groups with the problems. (Michael)

It allowed me to see the things that I still needed to work on and thanks to my peers, I was able to work on those things and be a better student and get better at those topics. (Felix)

I like to be able to explain my work and teach others because that helps me learn. (Maura)

I think it is helpful to have us teach it to our classmates like ... so that we can explain it to each other. (Julie)

Participants also described group work as a time for co-teachers to provide further support to students in a smaller setting if needed. While in groups, participants noted that their teachers would circulate the classroom, facilitating discussion and giving aid and assistance to students based on teachers' assessment of student understanding.

Sometimes the teachers would break us into groups and one teacher would work with one group and another teacher would work with another group. (Felix)

She puts us in groups of which group understands it and the other group she will sit down and she will help them. And every time you are struggling with something, they would go up to you and help you or they would put you in groups to help. So they were very helpful. (Julie)

He would work with us one on one or in small groups, whichever the students were most comfortable with. (Joshua)

Although participants found that teacher-led and student-led group work was a teaching practice that they preferred, three participants mentioned that because of the classroom management style of their co-teachers, group work was limited or did not exist at all. One participant shared that while his co-teachers did put the class in groups occasionally, students' opportunity to participate in group work was often cut short due to student misbehavior. As a result, students had to complete the work on their own.

> *They will put us in groups, but sometimes if they are talking too much or they see that they are not doing the work, then they will have us do it alone.* (Michael)

The other two participants shared that they wanted to engage in group work with their classmates, but their teachers limited any opportunity for them to do so.

> *We couldn't really talk to each other during class. So I couldn't help or explain it to my classmates. I tried when we had time, but we always had to work on our own and she would yell at us if we were talking. The teacher didn't really let us interact with each other. If I tried to explain it to someone, I would get in trouble for talking. Even ... like I would tell them that I was trying to help, but they wouldn't believe me, so I just tried to stop helping because I didn't want to get in trouble.* (Joshua)

> *I think group work would be really helpful so that I could work with my peers and I could hear how they solved it and they could hear how I solved it. In middle school, I remember when I would be in a group, we could help explain the problems to each other and have different perspectives and we had a better understanding as a group to see how we can understand the topic.* (Orlando)

Participants expressed their preference for co-teachers to incorporate small group work into daily instruction. In addition to receiving support from their teachers, participants described the importance of being given the time to share their understanding with their peers. During group work, participants wanted the chance to learn from their peers so that they could all better understand the topic through different perspectives. However, because co-teachers have different classroom management styles, group work was limited in some participants' classrooms.

Discussion

In this research study, participants provided rich data that helped to understand their experiences with mathematics instruction in their high school ICT classes. Because research that includes the voices of students with LD is limited, little was known about their thoughts, preferences, and overall assessment of their mathematics instruction within an ICT setting. In line with the work of Leafstedt et al. (2007), participants in this study were aware of the ways they wanted to be taught. In the current study, participants shared that they preferred when their teachers broke down new mathematical concepts using explicit instruction and provided opportunities for repeated exposure and guided practice. Furthermore, participants wanted their co-teachers to show them multiple ways to solve a problem. Not only did participants feel that the pace of instruction was rushed, but they also had to use their free time during or after the school day to seek extra help from their special education teachers. The findings from this study bring up issues at both an instructional and policy level that have not received much attention in the literature because the voices of students with LD are so often left out.

The participants in this study talked extensively about explicit instruction. While they did not reference the term explicit instruction directly, participants continued to mention “break down,” “step-by-step,” and “practice it," all terms that align with the concept of explicit instruction. These results are similar to the students with LD in Leafstedt et al.’s (2007) study, who also mentioned that they wished their work was broken down for them. To meet students’ needs, mathematics teachers can incorporate instances of explicit instruction such as modeling new mathematics material, and while doing so, breaking down content into individual steps (Doabler et al., 2012; Weibe Berry & Namsook, 2008). The results suggest that during explicit instruction, teachers may want to consider providing students with opportunities for guided practice. During guided practice, teachers may want to encourage students to share their understanding, ask questions, and seek clarification. Incorporating opportunities for participation makes the thinking of students visible, not only to the teacher but to all students in the class. As a result, teachers can provide valuable and timely feedback (Bonner & Chen, 2019), which was a teaching practice that participants in the current study preferred. Immediate feedback gives students a chance to gauge their understanding and self-regulate their learning before completing their group work or independent practice.

While participants shared that they preferred explicit instruction, it is essential to consider how students with LD perceive success in mathematics. For instance, do students with LD believe that success is their ability to follow a procedural approach? If so, one alternative explanation for the results of this study is that students with LD consider success as finding the correct answer using a procedure. Additionally, what does it mean when a student says that one particular instructional method helps them understand better than another? Another potential alternative explanation is that participants have only been exposed to explicit instruction, and as such, believe that they need it within their mathematics instruction to feel that they can be successful. Within this study, it is not clear the beliefs participants have about success. As such, more research is needed on the ways in which students with LD define and perceive achievement and success in mathematics.

The participants in this study were acutely aware that all students learn differently. As such, participants shared that they wanted their co-teachers to show them multiple ways to solve the same type of problem. Additionally, they wanted their co-teachers to be more flexible and responsive to how students learn. It is essential that mathematics co-teachers continuously collaborate and reflect on the instructional practices they commonly use within their daily instruction. Co-teachers may want to consider if any teaching practices are limiting students' understanding. Certain teaching practices may be excluding students with LD from accessing the standards-based curriculum, such as requiring students to use a specific mathematical approach. In this study, some participants shared that they had to take the initiative to teach themselves a mathematical concept when their teachers did not acknowledge their learning needs and misunderstandings. One way in which teachers can encourage the inclusion of all students is by utilizing multiple teaching strategies rather than relying on only a few. Additionally, a more inclusionary practice would be to ask students about their preferences (Cook-Sather, 2002), and identify and honor the cognitive strengths of students with LD. Planning instruction in a way that encourages students to use their prior knowledge to develop new mathematical knowledge (Lambert, 2018) is another inclusionary practice that teachers may want to consider. To better support co-teachers, more research on effective teaching practices for students with LD in algebra is needed. Once these strategies have been identified, they can inform curriculum design and pacing, as differentiation strategies are a crucial component of instructional planning and implementation.

The participants in this study described in detail that they felt the pace of instruction was rushed. Similar to those students with LD in Leafstedt et al.'s (2007) study, participants within the current study believed that the speed of daily lessons and the pace at which co-teachers moved from one topic to the next was too quick for them. Because of their co-teachers pacing through the curriculum, participants felt they did not have a chance to solidify their understanding. Overall, it was evident that participants wanted more time with the content, but why were they not afforded this opportunity? Furthermore, why was the pacing of instruction not meeting the needs of these students with LD? The premise of an ICT setting is that co-teachers work together to provide instruction to meet the needs of a wide range of ability levels in a classroom of students with and without disability labels with the goal of improving the performance of all students (Cook & Friend, 1995; Cook et al., 2017; Friend, 2008; Mastropieri et al., 2005). However, improving or increasing academic achievement in mathematics may be challenging if the pace of instruction feels hurried to students with LD. Furthermore, is the pacing for students without disability labels also insufficient? Why or why not? To better understand instructional pacing, there is a need for research that explores how a larger group of students with LD perceives the speed of instruction within their ICT mathematics classes. Additionally, research that seeks to identify ways in which co-teachers can appropriately pace their instruction within an ICT setting to better meet the needs of students with LD is imperative.

Within the classroom, co-teachers may want to give students the opportunity to share their thoughts about the speed of instruction and their understanding. One way that co-teachers can receive feedback about their instruction is by interviewing students (Mitra, 2003). However, a potentially more quick and private way of receiving student feedback is by giving students the chance to express and reflect on paper (Heritage, 2013). For instance, co-teachers can give students the opportunity to rate the speed of the lesson with options such as "too fast," "too slow," or "just right." Similarly, teachers can provide students the opportunity to rate their understanding of the topic with options such as "I do not understand," "I need some help," "I understand it," and "I can teach it." Additionally, co-teachers can gather evidence in an effort to monitor student understanding by assigning an exit ticket at the end of class (Heritage, 2013). To better gauge students' understanding, co-teachers may want to incorporate open-ended tasks, as they require students to show their work when solving, rather than circling an option on a multiple-choice question. By reviewing students' responses on the exit ticket, co-teachers can determine whether students grasped the lesson for that day. Co-teachers can use student feedback and exit ticket data to plan instruction for the following day (Bonner & Chen, 2019; Heritage, 2013). In some cases, co-teachers may want to quickly review a misunderstanding, whereas in other cases, co-teachers may determine that they need to spend another day on a concept.

Although data from exit tickets can be informative for co-teachers, external pressures may prohibit co-teachers from responding to data appropriately. Ideally, co-teachers can use data from exit tickets to modify the pacing of classroom instruction. However, teachers may feel pressure at the policy level to move on to the next concept before students are ready. The Common Core State Standards for Mathematics (CCSSM) were adopted in the state where this study was conducted. Toward the end of the school year, students are required to take a high-stakes standardized test that assesses students' mastery of the CCSSM. As such, teachers may feel the need to move from one concept to the next in order to teach the standards that will be assessed on the standardized test. The adoption of the CCSSM aims to ensure that all students throughout the entire state are learning the same content and have access to the same curriculum and standards. However, standards-based accountability that is often measured through high-stakes testing is an exclusionary practice in which the needs of some students are being ignored. In the current study, the pacing of instruction was too fast for students with LD. As such, participants in this study may not have been receiving the high-quality instruction that they deserve and that federal law mandates (IDEA, 2004). Educational policy and school administration put teachers in a difficult decision making spot. For instance, do teachers respond to students and slow down instruction to meet students' needs or do they continue to move through the curriculum? How do teachers' decisions potentially affect or limit students' understanding of future lessons and content? At an even more basic level, how do teachers make these decisions, and how might decisions differ from teacher to teacher and school to school? The decisions that co-teachers make when planning and executing instruction can have long-term effects on students with LD in mathematics, such as limiting their conceptual understanding and their opportunity to access higher-level mathematics courses. Further research is needed on how co-teachers make instructional decisions and respond to data in their effort to provide high-quality instruction to students with LD in ICT settings.

Although participants were not explicitly asked about extra help, five of the six participants in this study discussed the need to seek extra help outside of their mathematics class. Often, participants in this study, and those in Leafstedt et al.'s (2007) proactively sought out their special education teachers on their own time to finish their work, ask for work to be re-explained, and receive extra help and practice. In the current study, some participants even found other special education teachers in their school for support. While participants did not express any negative feelings toward taking ownership of their learning and seeking assistance outside of class, often during their free time, it does raise several questions as to why this practice is occurring so frequently. Additionally, this finding brings up concerns as to why students with LD seeking extra help on their own time is such a popular practice across schools, as all participants attended different high schools in the urban area where this study was conducted. For example, are students with LD receiving appropriate help and individualized attention during class? What factors are limiting students with LD from completing their work during class? Is the rate that students with LD seek extra help outside of class similar or comparable to that of their peers without disability labels? Because students with LD must use their time at lunch or after school to meet with their teachers, are they excluded from opportunities to participate in other activities or socialize with their peers? Finally, what happens to students with LD if they do not proactively meet with their teachers to complete their work or receive extra help? Students with LD may receive lower grades, leading to course failure (Cortiella & Horowitz, 2014) because their work is incomplete, or they may have difficulty developing their understanding of the content. Due to their incomplete understanding, students with LD may fall behind the pacing of the curriculum and standards. If students do not pass the high-stakes standardized test at the end of the school year, they may need to either repeat the class or retake the test until they achieve a passing grade. Based on these lingering questions and potential implications for students with LD, there is a need to explore and understand how ICT mathematics classes are structured and how students with LD are supported within these settings.

Limitations and Implications

The purpose of this study was to explore the perceptions of students with LD regarding their mathematics instruction within an ICT class. Because of the limited number of participants, the results must be interpreted with caution. Furthermore, these qualitative findings cannot be generalized beyond these participants but should be transferred to other similar contexts. Although semi-structured interviews provided many opportunities for student voice, data were self-reported by participants. Therefore, data were limited to what participants decided to disclose and not disclose to the researcher during the time of the interview. At the time the study was conducted, the pandemic caused by COVID-19 forced school closures throughout the United States. As a result, all participants received remote instruction for approximately three months of the 2019-2020 school year. Because the interviews were conducted in the summer of 2020, this experience may have influenced participants' responses about their mathematics instruction. Additionally, rather than being conducted in person, as previously planned, all interviews were conducted virtually through Zoom. This platform may have also influenced participation.

Further research on which teaching practices co-teachers are incorporating within their ICT mathematics classes is needed. Because explicit instruction was the most discussed and preferred practice, it is unclear whether participants in this study have been exposed to practices such as inquiry-based instruction. As such, participants in this study may have preferred explicit instruction because they have grown accustomed to it over the course of their educational experience. Explicit instruction seems to be a common practice utilized for mathematics instruction, as all participants in this study attended separate high schools with different teachers throughout the urban area where this study was conducted. Therefore, examining other instructional practices that align with a standards-based curriculum within an ICT class and how students with LD perceive these instructional practices is an area of additional research. Research in this area can help identify other instructional components that students with LD perceive as advantageous or disadvantageous.

Even though this study presents several limitations, it does highlight the need to give voice to students with LD regarding their educational experiences and instructional preferences. Participants shared that explicit instruction was the primary mode of instruction within their ICT classes. While participants noted that they preferred explicit instruction, they described various ways in which their teachers did not meet their learning needs. Participants highlighted that explaining the content in multiple ways, repeated exposure, appropriate pacing, and group work would help them develop mastery of mathematics content better. To meet the needs of students with LD, further research on inclusionary teaching practices within ICT mathematics classes is essential. Furthermore, there is a need to study and re-examine the ways in which standards-based accountability is potentially excluding students with LD from receiving high-quality instruction. Most importantly, it is evident that students with LD are able to articulate their learning needs and preferences. As such, it is imperative that school administrators and educators do a better job giving a voice to their students. Teachers can administer learning preference surveys, multiple intelligence assessments, or simply conduct interviews with students at the conclusion of a lesson to receive feedback. Collaborating with students to receive their input regularly can help teachers determine appropriate instructional strategies and pedagogy. Furthermore, by giving voice to students, teachers can consider students' preferences when modifying the curriculum and planning future lessons and units.

CHAPTER III

DEMONSTRATING THEIR KNOWLEDGE AND UNDERSTANDING OF LINEAR FUNCTIONS: WHAT DO STUDENTS WITH LD KNOW?

Abstract

To design effective mathematics instruction for students with a learning disability (LD), teachers may wish to consider students' previous conceptions and misconceptions. However, little is known about the ways that students with LD think about linear functions. In the present study, six high school students with LD participated in a mathematical task interview, also known as a clinical interview. While completing a series of tasks on linear functions represented in different ways, participants shared their thinking while finding the rate of change and y-intercept. Both participants' mathematical work and the explanations of their work were analyzed. Results indicate that participants showed an emergent or procedural understanding of linear functions based on the manner in which they approached each task. There was little evidence of participants using methods that would suggest a conceptual understanding of the rate of change or y-intercept. While five of the six participants demonstrated that they could recall a valid procedural approach to find the rate of change and y-intercept, participants had a more difficult time executing these procedures with precision and accuracy. Participants' procedural understanding of the rate of change and y-intercept were categorized into the following levels: (a) novice, (b) developing, or (c) proficient.

There have been ongoing efforts from the National Council of Teachers of Mathematics (NCTM) to reform mathematics education in the United States by increasing academic rigor. Rather than memorizing facts and procedures, the NCTM encourages teachers to push students to build a conceptual understanding while simultaneously developing students' logic and reasoning skills (Jitendra, 2013; Montague et al., 2008). Students in the United States have shown growth in mathematics on the National Assessment of Educational Progress (NAEP) overall (Watt et al., 2016). However, compared to their peers without disability labels, students with a learning disability (LD) label still earn lower mathematics scores on the NAEP, particularly on the algebra subtest (Cortiella & Horowitz, 2014; Watt et al., 2016).

To support students with LD in mathematics, there is a need to understand students' content knowledge in algebra, as both Algebra I and Algebra II are required for high school graduation (NCTM, 2014; USDOE, 2010). Linear functions is an algebraic topic typically introduced in middle school, and it continues to appear throughout higher levels of mathematics (Teuscher & Reys, 2010). Not only is an understanding of linear functions foundational for algebraic thinking (Beatty & Bruce, 2012), but also Dubinsky (1993) explained, "It can be argued that functions form the single most important idea in all mathematics, at least in terms of understanding the subject as well as for using it" (p. 527). In their work, Capraro and Joffrion (2006) stated that an "understanding of linear equations and algebraic relationships is fundamental to preparing students for success in advanced algebraic concepts" (p. 147). However, both essential concepts of a linear function, the rate of change (Herbert & Pierce, 2012; Teuscher & Reys, 2010; Wilkie & Ayalon, 2018) and the y-intercept (Davis, 2007; Knuth, 2000), are often problematic for students with and without disability labels to understand conceptually. Students may struggle with these concepts because they tend to rely on certain algebraic procedures (Zahner, 2015) rather than developing and utilizing a conceptual understanding of functions (Capraro & Joffrion, 2006).

Rate of Change

The rate of change is one of the two key concepts of linear functions. Ayalon et al. (2015) defined the rate of change as “a relationship in which changes in one variable can be expressed formally or numerically in terms of changes in another variable” (p. 323). In the early years of algebra or pre-algebra, teachers present the rate of change as the slope or steepness of a line. Often, teachers and students use these terms interchangeably (Teuscher & Reys, 2010). Researchers have studied the ways in which students without disability labels use the correspondence approach and the covariation approach to find the rate of change. Like the input-output model of a function, in the correspondence approach, students develop a rule that allows them to solve for any value of *y* based on the value of *x*. On the other hand, the covariation approach “involves analyzing, manipulating, and comprehending the relationship between changing quantities” (Ayalon et al., 2015, p. 323). In their work, Ayalon et al. (2015) found that a higher percentage of students in Years 7 to 13 (i.e., US grades 6 to 12) in the United Kingdom used a correspondence approach compared to a covariation approach. Those students who used a covariation approach were more successful in accurately developing a sequential rule to complete the task. However, Wilkie and Ayalon (2018) found that students ranging from Years 7 through 12 (i.e., US grades 7 to 12) in Australia favored and were more successful utilizing the correspondence approach than the covariation approach.

To find the rate of change of a linear function, students may resort to using a procedural or pattern approach. When studying a ninth-grade bilingual algebra class, Zahner (2015) found that students utilized a procedural approach to find the rate of change of a linear function.

Although students could accurately find the rate of change using a computational procedure, they could not explain the meaning of the rate of change in terms of the context of the problem. For example, students described the rate of change as the "rise over run" or the "up and over." Rather than developing an understanding of the underlying mathematical concept, students simply knew how to complete the steps of a procedure, which suggests that they understand functions at a superficial level (Kieran, 1992).

Although finding the rate of change can be a difficult concept for students to grasp, an emphasis on quantities and how quantities relate to each other can help support students' development. In her work, Ellis (2009) studied a seventh-grade mathematics class that focused on real-world quantities and an eighth-grade mathematics class that focused on patterns in a number table. Because the eighth-grade teacher taught students primarily using a table, many of their patterns referenced the columns of the table separately. For example, students noted that as x increased by one, the other side went up by seven. These students did not make a connection or establish a relationship between the independent and dependent variables. On the other hand, the seventh-grade students made better sense of linear functions such as $y = mx + b$ situations because of their focus on quantities throughout the school year. They made accurate generalizations and global rules, and they explained and supported their ideas.

Although students learn about the rate of change as early as middle school, they can develop incomplete or inaccurate understandings that have the potential to stay with them as they continue their education. In their work, Teuscher and Reys (2010) studied students in Advanced Placement (AP) calculus ($n = 191$). On a pre-assessment administered at the beginning of the school year, students showed that they had not yet mastered the concepts of slope, rate of change, and steepness of a graphical representation, and they did not understand the relationship between these three concepts. When given a graphical representation of a function, students had a difficult time understanding that the sign of the slope was meaningful. For example, some students did not acknowledge the fact that a slope of -2 was different from a slope of +2. Although these students took math courses to prepare them for AP calculus, it was evident that they did not have a solid foundation of a mathematical concept that they would need to understand more abstract concepts such as derivatives.

Y-Intercept

Most textbooks refer to the y-intercept as the value of the y-coordinate when its x-coordinate is zero or the value of the y-coordinate when the linear function crosses the y-axis on a graphical representation (Knuth, 2000). In his study of eight high school students, Davis (2007) found that students interchanged the terms *start* and *y-intercept.* When referring to a table, 16 of 17 utterances were *start.* However, the use of the term *start* in these situations was not an accurate referral to the y-intercept. When asked to define the "starting point," students shared responses such as the place where the function began, the value of *y* when *x* was equal to zero, or the place where the function crossed the y-axis on a graph. While some students could accurately define the y-intercept, they struggled to find the y-intercept from a table or an algebraic equation.

Students have trouble recognizing that the y-intercept is a fixed amount. In their work, Pierce et al. (2010) studied 15-year-old students (n = 70) in Australia and found that students were more concerned about the rate of change than the y-intercept. When asked how much it would cost to hire a plumber who charged an initial hiring fee and a constant rate per hour of work, students often focused on how much it would cost based on the number of hours the plumber worked. Students did not refer to the y-intercept in their explanation. On the other hand, when asked to interpret the meaning of the y-intercept from a graph, some students did reference the y-intercept. However, they considered the y-intercept the starting value when $x = 1$ rather than when $x = 0$. Students shared their interpretation of the graph as “start at $75 for the first hour and then just keep adding $50” (Pierce et al., 2010, p. 209). Similarly, Hattikudur et al. (2012) found that middle school students did not graph the y-intercept on the y-axis. Other students disregarded the given y-intercept and, instead, graphed the y-intercept at the origin of the coordinate plane. These misconceptions about the y-intercept are problematic because, to students, the y-intercept “is often seen as an accessory to the function, rather than a vital part of it” (Pierce et al., 2010, p. 212).

Multiple Representations

When working with functions, students may use or alternate between representations, but this does not necessarily mean that they have fully developed their understanding of functions. In their work, Adu-Gyamfi and Bosse (2014) studied eight high school students in a pre-calculus class in which instruction emphasized the use of multiple representations of functions when problem-solving. While students showed the use of alternative mathematical representations such as tables, graphical representations, and algebraic equations, Adu-Gyamfi and Bosse found that students transitioned between representations depending on the problem and their interpretation of the task. In addition, when deciding which representation students would use to solve the problem, Adu-Gyamfi and Bosse found irregularities with student reasoning and common and uncommon rationales for using selected representations. Similarly, Filloy and Sutherland (1996) suggested that students’ current stage of development might influence their use of different approaches and representations.

If given the option, students tend to rely on algebraic equations. Of the 178 students in high school classes ranging from pre-algebra to calculus, 75% of students used an algebraic approach as their primary method of solving functions, even when a different representation, such as a graphical approach, would have been easier (Knuth, 2000). In fact, Knuth (2000) noted that tasks in his study were designed in an effort to drive students to use a graphical representation. However, less than one-third of the participants used a graphical representation as their primary or alternative method. Many students did not even acknowledge that they could utilize a graphical representation. Furthermore, Knuth noted, "students failed to recognize or create the connection in the graph-to-equation direction" (Knuth, 2000, p. 503). Knuth's findings support Hart's (1981) work in that both found it was difficult for students to make a connection between ordered pairs on a graph and an algebraic equation.

Conceptualizing Linear Functions for Students with LD

Issues in developing conceptual understanding of algebraic concepts such as linear functions may arise for students with LD because of the level of abstract thinking that algebra entails (Witzel et al., 2003). Rather than portraying mathematics in pictures or using concrete representations, algebra requires students to recognize and manipulate symbols and understand numerical relationships and mathematical structures (Linsell, 2009). Furthermore, students with LD usually rely on memorizing facts and procedures (Capraro & Joffrion, 2006), or they resort to utilizing guess and check methods (Herscovics & Linchevski, 1994). To design instruction to meet the needs of students with LD, it is vital to give voice to their knowledge of linear functions. However, the research presented above focused on students without disability labels. In fact, little research on this topic includes students with LD, and the limited mathematics education research that does include students with LD focuses on elementary classroom instruction and concepts (Beatty & Bruce, 2012; Geary et al., 2008; Gersten et al., 2009). While high school students with LD may experience similar struggles to students without disabilities, there is a need to know the ways in which students with LD think about linear functions. Revealing the content knowledge of students with LD can assist teachers as they develop and provide high-quality, rigorous instruction to students with LD. In an attempt to begin to fill this gap in the literature, the purpose of this study is to address the following research questions:

1. What conceptions of linear functions do students with LD possess as evident in their work on problems with abstract graphical representations and real-world connections?
2. Based on existing literature, to what extent, if any, does the way in which students with LD approach tasks on linear functions differ from students without disability labels?

Method

An interview study was conducted with high school students with LD. To address the research questions, each participant completed a series of three mathematical tasks.

Participants

This study was conducted with students with LD that attended a public high school in a large urban school district in the northeast United States at the time of the study. All participants met the following criteria: (a) were enrolled in ninth, 10^{th}, 11^{th}, or 12^{th} grade at a large urban high school; (b) were enrolled in an Integrated Co-Taught (ICT) class for mathematics; (c) had an Individualized Education Program (IEP); (d) self-identified as having a learning disability; and (e) gave student assent and obtained parent or guardian permission to participate. The researcher recruited participants by posting flyers at local community centers and snowball sampling. Six participants were included in this study. Rather than only including students with mathematics disabilities, any student with LD was eligible to participate because the urban area in which this study was conducted does not differentiate between the types of LD on a student's IEP. In addition, because Watt et al. (2016) reviewed 15 studies of students with LD in algebra and found that only 13% of participants were Hispanic and 44% were Black, the researcher gave additional effort to recruiting students who identified as Black or Latinx to gain a more diverse perspective.[2] However, this was not a requirement to participate. All six participants in this study attended schools where 100% of students received free or reduced lunch. The demographic characteristics of each participant in this study at the time of the interview are shown in Table 2.

Table 2

Participant Demographics at Time of Interview

Pseudonym	Sex	Age	Grade	Race/Ethnicity
Julie	Female	17	12	Black
Joshua	Male	17	12	Latinx
Felix	Male	16	11	Black
Michael	Male	16	11	Latinx
Orlando	Male	17	11	Black
Maura	Female	15	10	Black

Note. Demographics self-identified by participants.

Data Collection

This study was part of a larger study in which participants completed a two-part interview with the researcher. The first part of the interview was a semi-structured interview, which explored participants' perceptions of their mathematics instruction within an ICT setting and results are reported elsewhere. The second part of the interview was a mathematical task

[2] Cited as Hispanic in Watt et al.'s (2016) study

interview, also known as a clinical interview (Clement, 2000), and comprise the data used in this study. Mathematical task interviews were conducted over a two-month period between July and August. All interviews were conducted by the researcher immediately following the semi-structured interview and lasted approximately 30 to 45 minutes. Due to the COVID-19 pandemic, the researcher conducted all mathematical task interviews virtually through Zoom. The researcher utilized PearDeck, a Google Slides Add On, which allowed the researcher to see participants' work virtually, in real-time. The mathematical task interviews were audio recorded and transcribed by the researcher.

During the mathematical task interview, participants completed a series of three tasks with a specific focus on linear functions. Two tasks were adapted from the Mathematics Assessment Resource Service (MARS; Mathematics Assessment Project, 2015), and one task was a short-response question from the state assessment in high school algebra. Although students completed three tasks during the mathematical task interview, the results from only two of the tasks were included in this study. These tasks were included because in both tasks participants had to write a linear equation by finding the rate of change and y-intercept. To better grasp participants' understanding of linear functions, a real-world task was included to encourage students to use prior knowledge in their attempt to complete tasks on abstract mathematical concepts such as linear equations (Davis, 2007; Leinhardt et al., 1990). Additionally, to see if participants could explain the meaning of the rate of change and y-intercept in terms of the context of the problem (Pierce et al., 2010), a table of values based on a real-world problem was incorporated. An abstract graphical representation was included because students' prior knowledge of real-world problems may lead them to misinterpret linear equations

(Davis, 2007). Appendix G notes how each concept, the rate of change, the y-intercept, and multiple representations, aligned with each task.

During interviews, the researcher read each task aloud to ensure that language and reading comprehension did not influence students' access to the tasks. The researcher prompted participants to explain their actions and thoughts out loud (Lewis & Fisher, 2018) by asking questions such as "How did you solve that?" and "Tell me more about what you are thinking." Furthermore, to gain a deeper understanding of students' internal cognitive processes, the researcher encouraged students to justify their solution verbally, possibly by using a different method, and to explain their thought process (Hunt & Empson, 2015; Lewis & Fisher, 2018). The purpose of the mathematical task interview was not to test the accuracy of students' work (Hunting, 1997), but rather to understand students' underlying thoughts about a concept (Goldin, 2000) and the ways in which they verbally, symbolically, and pictorially represent mathematical tasks (Hunt & Empson, 2015).

Mathematics education researchers and teachers have used mathematical task interviews as a tool to design appropriate instruction to meet the needs of students. The Individuals with Disabilities Education Act (IDEA; 2004) requires that teachers deliver specially designed instruction to students with disabilities based on their strengths and needs (Lewis & Fisher, 2018). To provide appropriate instruction, teachers may want to consider assessing students' knowledge rather than making assumptions about students' level of understanding based on their educational trajectory. Not only do mathematical task interviews allow for the investigation of students' mathematical thinking (Ulusoy & Argun, 2019), but they also "can give more information on depth of conceptual understanding, since oral and graphical explanations can be collected, and clarifications can be sought where appropriate" (Clement, 2000, pp. 1-2). In his work, Clement (2000) explained that an interview allows the investigator the opportunity to react and respond to data being collected at the moment. Based on data, the investigator has the opportunity to ask new or clarifying questions to gain more insight into naturally hidden thought processes. To support students with LD, mathematics education researchers and teachers can use mathematical task interviews to identify essential learning opportunities and align instruction accordingly.

Data Analysis

Data analysis was informed by the research questions. All mathematical task interviews were recorded and transcribed. In addition, participants' mathematical work was collected and included as part of data analysis. Provisional coding was used to code each transcript. The researcher generated 12 provisional codes based on existing literature. As the literature on students with LD is limited in this field, the provisional codes were derived from existing studies on students without disabilities. During the first round of coding, the researcher went line by line of each transcript using the provisional codes. Next, based on the type of mathematical understanding that the provisional code would suggest, the researcher examined and organized the provisional codes into the following three themes: (a) emergent understanding, (b) procedural understanding, or (c) conceptual understanding. Participants that approached the mathematical tasks using irrelevant or invalid approaches showed an emergent understanding of linear functions. In this case, participants demonstrated little evidence of using procedural or conceptual approaches. Because a procedural understanding refers to students' use and knowledge of mathematical language, symbols, rules, and algorithms (Capraro & Joffrion, 2006), participants that followed step-by-step procedures demonstrated a procedural understanding. In terms of linear functions, participants that utilized the slope formula or a pattern approach showed a procedural understanding (Teuscher & Reys, 2010; Zahner, 2015). Finally, a conceptual understanding refers to students building knowledge that is rich in relationships by linking new ideas to their previous conceptions (Stump, 2001). One example of a conceptual understanding would be the use of the covariation approach to develop accurate global rules of linear functions (Ayalon et al., 2015). Table 3 shows how some provisional codes were sorted into themes.

Table 3

Alignment of Provisional Codes with Themes

Provisional Code	Theme	Example
Covariation	Conceptual Understanding	Participant mentions the relationship between the amount spent, in dollars, it took Tanya to make a various number of cards in Task 3
Correspondence	Procedural Understanding	Participant develops a pattern that they could have used to solve for the amount Tanya spent based on a certain number of cards
Slope Formula	Procedural Understanding	Participant substitutes the values of two coordinate pairs into the slope formula

Finally, within these three themes, the researcher reviewed and analyzed participants' mathematical work in terms of the accuracy of their reasoning and their solution. The researcher analyzed participants' work on finding the rate of change and the y-intercept separately. Based on their work and their solution, the researcher classified participants into the following categories: (a) novice, (b) developing, or (c) proficient. A participant that recalled and attempted to use a valid procedure to find the rate of change or y-intercept was a novice within the theme of procedural understanding. A procedural novice differs from a participant with an emergent understanding. Rather than rather referencing, recalling, or using an approach that would indicate a procedural understanding of linear functions, such as the slope formula, a participant with an emergent understanding used an invalid or inappropriate method to write a linear equation. Table 4 provides an example for each level of skill under the theme of procedural understanding.

Table 4

Levels of Procedural Understanding Finding the Rate of Change.

Theme	Level	Example
Procedural	Novice	Identifies appropriate procedure but does not apply it correctly
Procedural	Developing	Identifies and applies appropriate procedure with some accuracy but not consistent across both tasks
Procedural	Proficient	Identifies and applies appropriate procedure with accuracy and consistency across both tasks

Because the researcher analyzed students' work on the rate of change and y-intercept separately, it was possible for a participant to be proficient in finding the rate of change, but a novice when solving for the y-intercept.

Mathematical Tasks

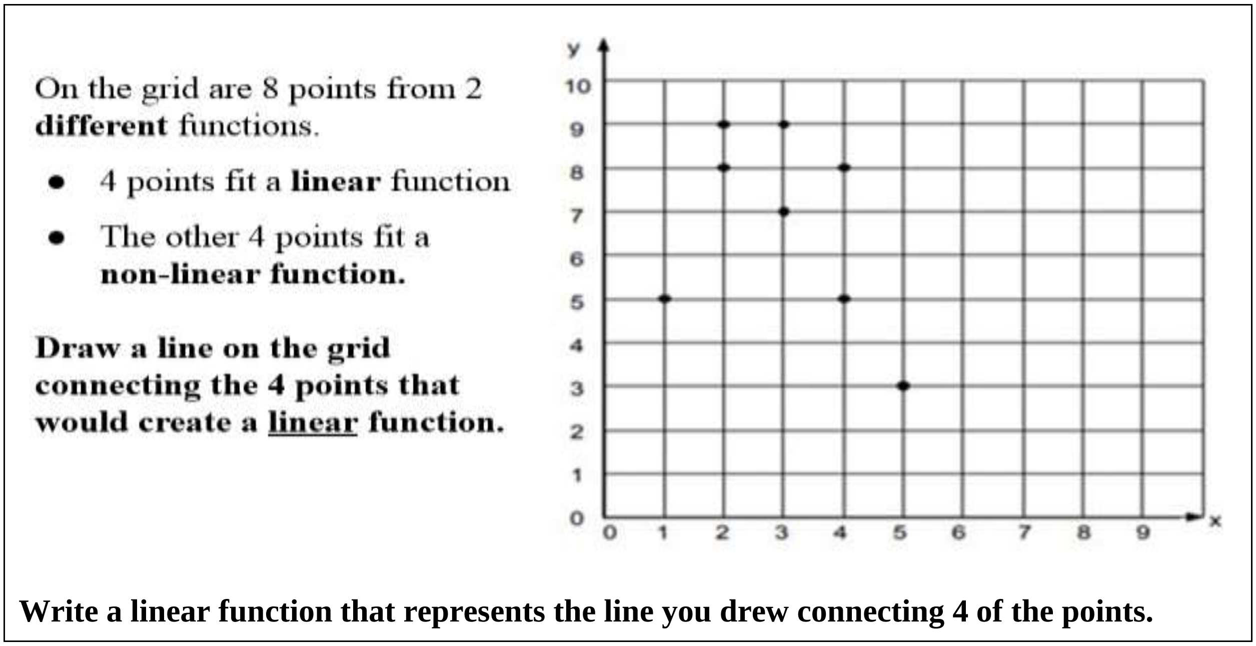
Mathematical task interviews were conducted with participants. Each interview consisted of three different tasks, all of which were related to linear functions. Only the results from two of the three tasks are included within the current study.

Task 1

Task 1 was adapted from another MARS resource (Mathematics Assessment Project, 2015). In this task, students were given the first quadrant of a coordinate plane with eight graphed ordered pairs. Four of the points created a linear function, while the other four points represented a nonlinear function. To create a linear function, participants had to select and connect four of the ordered pairs. Then, participants had to write an algebraic equation to represent the function. Figure 1 shows the abstract graphical representation given to participants in Task 1.

Figure 1

Mathematical Task Interview, Task 1



While the original MARS task asked students to identify the ordered pairs on the linear function, this was removed for the purpose of this interview. The rationale for this decision was that identifying ordered pairs may potentially encourage participants to use a procedural approach rather than looking at the graph globally to determine the rate of change and y-intercept.

Task 2

The researcher adapted the second task from a question that was given on a previous statewide assessment in high school algebra administered in the state where this study was conducted. In addition to writing a linear equation based on the table of values that represented a real-world scenario, participants were asked to explain the meaning of the rate of change and the y-intercept in relation to the context of the problem. Figure 2 presents the real-world context and table of values given in Task 2.

Figure 2

Mathematical Task Interview, Task 2

Tanya is making homemade greeting cards. The data table below represents the amount she spends in dollars, $f(x)$, in terms of the number of cards she makes, x.

Number of Cards, x	Amount Spent in Dollars, $f(x)$
4	7.50
6	9
9	11.25
10	12

Write a linear equation to represent the function.

Results

Based on the manner in which participants approached the tasks, their understanding was organized into one of the following themes: (a) emergent understanding, (b) procedural understanding, or (c) conceptual understanding. Each level of understanding is described in detail.

Emergent Understanding

Of the six participants, only one participant in this study demonstrated an emergent understanding of linear equations. In Task 1, Orlando correctly identified only three of the four ordered pairs. When asked why he selected the incorrect ordered pair, Orlando shared that "(2, 8) would make the line straight." Orlando did not mention or attempt to find a pattern from one coordinate to the next or prove that including the point (2, 8) would make a straight line. In the next part of Task 1, Orlando had to find the rate of change. During this part of the task, Orlando did not identify or apply an appropriate approach to find the rate of change. Instead, Orlando focused on two ordered pairs on the plane, (1, 5) and (2, 8). Then, Orlando shared that the slope is 8 and the y-intercept is 5 because "the starting point is 5 and the ending is 8." When probed further, Orlando could not clearly explain his rationale for the rate of change. Rather, he continued to focus on the two ordered pairs that he selected. Similarly, in Task 2, Orlando selected only one ordered pair from the table to write an equation. Orlando circled the pair (6, 9) on the table. Then, Orlando wrote that the equation as 6x = 9, which was incorrect. Orlando's work in Task 1 and Task 2 demonstrates one issue with relying on procedural approaches. Although Orlando knew that he had to use the ordered pairs to write an equation, he struggled to recall what he needed to do with the coordinates. Instead of recalling and applying an appropriate procedure on this task, he used the values of the coordinates that he selected in his effort to write an equation.

Procedural Understanding

When looking across tasks, most participants relied on using procedural approaches when working with linear functions. Participants utilized the slope formula, pattern approaches, and substitution to find the rate of change and y-intercept of linear functions.

Rate of Change

The most popular procedural approach used by participants to find the rate of change was the slope formula. Four of the six participants attempted to use the slope formula in Task 1 and Task 2. The slope formula is $\frac{y2-y1}{x2-x1}$, which consists of finding the difference between the y-coordinate values and the difference between the x-coordinate values. Participants' skills in using procedures to find the rate of change varied and are presented below in the following levels: (a) novice, (b) developing, and (c) proficient.

Novice.

One participant was a novice in applying procedural approaches because she had a difficult time utilizing the slope formula accurately. During her interview, Maura could identify the slope formula, as she wrote the formula correctly. However, in both Task 1 and Task 2, Maura struggled to substitute coordinate pairs into the slope formula to find the rate of change. In Task 1, Maura accurately identified the four coordinate pairs that would make a linear function. However, Maura did not use any of these ordered pairs to find the rate of change. In addition, she made errors when substituting the values of the ordered pairs that she selected into the slope formula. This was also evident in her work on Task 2. Based on her work in Task 1 and Task 2, Maura showed a limited understanding of how to apply the slope formula to find the rate of change. Table 5 shows the values that each participant found for the rate of change in Task 1 and Task 2.

Table 5

Participant Generated Rate of Change for Task 1 and Task 2

Pseudonym	Task 1 m = -2	Task 2 m = 0.75	Understanding	Level
Orlando	m = 8	m = 6	Emergent	NA
Maura	m -5	m = 1.16	Procedural	Novice
Julie	m = - ½	m = 2	Procedural	Developing
Felix	m = 2	m = 0.75*	Procedural	Developing
Joshua	m = -2*	m = 0.75*	Procedural	Proficient
Michael	m = -2*	m = 0.75*	Procedural	Proficient

Note. * indicates that the participant found the correct value for the rate of change

Developing.

Two participants showed that they were still developing their skills in applying procedural approaches to find the rate of change. In Task 1, Felix utilized the slope formula, but he substituted incorrectly. Because of this mistake, Felix found that the rate of change was 2 instead of -2. However, in Task 2, Felix accurately found the rate of change using the slope formula. Below is Felix's explanation of the procedure he took to find the rate of change.

> **Felix**: Okay so in this case the two points would be (6, 9) and (10, 12). So, 12 minus 9 is 3 and 10 minus 6 which is 4. So, the slope is 3 over 4. I could divide and that is 0.75. So y = 0.75x + b.

In this task, Felix did correctly substitute all values from the coordinate pairs into the slope formula. Although Felix accurately found the rate of change in Task 2, his attempts to utilize procedural approaches were inconsistent and, as such, his skills were still developing.

Another participant used a pattern approach to find the rate of change. Rather than substituting coordinate pairs into a formula, Julie sought to find a pattern from one ordered pair to the next. In Task 1, Julie shared that she "counted the boxes" to find the rate of change. With that said, Julie identified the rate of change as – ½ instead of -2. Julie mentioned that she "went over and up" as she was counting, which caused her to confuse the relationship between the variables. Similar to her approach in Task 1, in Task 2, Julie attempted to find a pattern in the table of values. Julie's thoughts about finding the rate of change in Task 2 are shared below.

> **Julie**: The number of cards is going up by 2 from 4 to 6 and then the amount spent in dollars is going up by ... From 7.50 and 9 it goes by 1.50.
>
> **Researcher**: Why do you think that?
>
> **Julie**: Because I was thinking about how I would graph it. So for the y-axis, it would be

the amount spent in dollars and the x-axis would be the number of cards.

Researcher: Okay so what is the rate of change?

Julie: The slope is 2.

Julie demonstrated that she was still developing her skills and procedural understanding because she identified appropriate procedures to utilize but found an incorrect value for the rate of change in Task 1 and Task 2. Utilizing procedural approaches can be problematic for students with LD, as it requires them to recall and apply a series of steps from memory. Julie and Felix recalled valid procedural approaches, but they did not utilize them successfully on tasks that were represented in different ways.

Proficient.

Two participants were successful using the slope formula to find the rate of change in Task 1 and Task 2. Participants appropriately selected ordered pairs from the linear function and accurately substituted the values into the slope formula. Joshua and Michael's explanations about finding the rate of change in Task 1 are shown below

Joshua: First, I would find the slope. I would find 2 points. And I would do y_2 minus y_1 over x_2 minus x_1. It is 7 - 9 over 3 - 2. 7 minus 9 is -2 and 3 – 2 is 1. -2 over 1 is -2.

Michael: I made a mistake and did the y first. I did (9, 2) instead of (2, 9). So that would be 7 – 9 over 3 – 2. So that would be … 7 minus 9 is -2 over 1, which is -2.

Similar to their work in Task 1, Joshua and Michael used the same approach to find the rate of change in Task 2. While Joshua and Michael found the value of the slope as the fraction of ¾ for Task 2, they both explained that they divided the fraction to find the value of 0.75 as the rate of change. During their interview, Joshua and Michael identified and applied a valid procedure to correctly find the rate of change of two linear functions, each represented in a different way.

Using procedural methods can lead to a superficial understanding of the rate of change. Not only does relying on procedural approaches lead to memorization and calculation errors, but it also can be problematic for students when trying to create global rules or applying this procedure on tasks presented in different forms. Additionally, participants at the developing and proficient level recognized that they could select any two ordered pairs from the given representation to find the rate of change. However, after participants used the slope formula to find a ratio written in fraction form, they divided to find the rate of change as a decimal. For example, in Task 2, by dividing, participants found a single value as the rate of change, such as 0.75. Identifying the rate of change as a single value may be problematic for students when explaining the relationship between the two quantities that make up the rate of change, as there is only one visible quantity in their answer.

Y-Intercept

Participants' efforts to find the y-intercept are presented below based on the following levels: (a) novice, (b) developing, and (c) proficient.

Novice.

Similar to her attempt to find the rate of change, Maura selected an appropriate procedure to find the y-intercept but failed to follow out the procedure correctly. In Task 1 and Task 2, Maura used substitution with an algebraic equation to find the y-intercept. Although Maura found an incorrect value for the rate of change in Task 1 and Task 2, she correctly substituted the value she found for the rate of change into the formula for a linear equation. Additionally, Maura knew that she needed to select and substitute the x and y values from one ordered pair into the linear function to find the y-intercept. However, Maura began to make errors when substituting and solving the equation. She had difficulty applying the appropriate procedures to solve for the variable in the equation, such as isolating the variable. As a result, even based on her rate of change, Maura did not find the correct y-intercept for either task.

Developing.

Four participants demonstrated a developing level when using procedures to find the y-intercept. When solving for the y-intercept Julie, Joshua, Michael, and Felix correctly found the y-intercept in one task, but not the other. The y-intercept that participants derived based on the task can be found in Table 6.

Table 6

Participant Generated Y-Intercept for Task 1 and Task 2

Pseudonym	Task 1 b = 13	Task 2 b = 4.5	Understanding	Level
Orlando	b = 5	b = 9	Emergent	NA
Maura	b = -44	b = -0.4	Procedural	Novice
Julie	b = 13*	b = 2.5	Procedural	Developing
Felix	b = 5	b = 4.5*	Procedural	Developing
Joshua	b = 11	b = 4.5*	Procedural	Developing
Michael	b = 13*	b = 19	Procedural	Developing

Note. * indicates that the participant found a correct value for the y-intercept

Joshua, Michael, and Felix relied on a procedural approach that required them to substitute and solve an algebraic equation to find the y-intercept. Below describes the steps that Michael shared when solving for the y-intercept.

> **Michael**: I would take a point, and I would put it into the equation. I would take the (3, 7) and I would put the 3 in the spot of the x and the 7 in for y. I would multiply -2 times 3, which gives me -6. I would re-write it as 7 = -6 + b. Then I would add 6 to both sides. So the y-intercept is 13.

Michael's explanation is similar to the responses that Joshua and Felix shared. In Task 1, Joshua and Felix incorrectly found the y-intercept. When solving, they did not attempt to graph the y-intercept to see if it made sense as part of the linear function. Instead, once Joshua and Felix found the y-intercept, they simply rewrote the equation and shared that they were finished with the task. Rather than using a conceptual understanding to share how they could find the y-intercept using another representation, Joshua, Felix, and Michael simply shared the steps they took to substitute and solve an algebraic equation to find the y-intercept.

On the other hand, Julie was the only participant that referred to the graphical representation to find the y-intercept in Task 1. Julie extended the linear function by following the pattern on the graph until she reached the y-axis. In doing so, Julie correctly determined that the y-intercept was 13. In Task 2, Julie attempted to use the pattern that she found in the table to work backward to find the y-intercept. Julie determined that for zero cards, the amount of money spent by Tanya would be $2.50. In her work, Julie miscalculated the value for the rate of change; as such, her attempt to continue the pattern to find the y-intercept was unsuccessful. Julie explained her procedure for finding the y-intercept below.

Julie: So my table I put that 2 cards, they will spend 5.00.

Researcher: What about for 0 cards?

Julie: I got 2.50.

Researcher: How did you figure that out?

Julie: Because when I knew that 7.50 and 9 goes by 2.50.

Regardless of if, their rate of change or y-intercept was correct, all participants were asked verbally to state the value of the rate of change and the y-intercept. Based on the value that they shared, all four of these participants that used procedural approaches could substitute their values for the rate of change and y-intercept into an equation in slope-intercept form.

Proficient.

In this study, none of the six participants correctly found the y-intercept for both Task 1 and Task 2. Whichever procedural approach participants used in Task 1, they used the same approach in Task 2. However, when applying the procedural approach, participants struggled to attend to precision and use the procedure accurately across both tasks. Participants' errors in their work show the potential challenges of relying on procedural approaches to find the y-intercept. Participants struggled to recall and apply the steps of a procedure to find the y-intercept. In addition, participants made errors when substituting and when performing mathematical calculations. A limited understanding of the y-intercept can be problematic for students as they try to problem-solve with different representations of linear functions. With a deeper conceptual understanding, students can approach tasks using any representation and justify their solution based on their knowledge. When students build their conceptual understanding, they possess the logic and reasoning as to why various procedures can help to solve the task. However, with just a procedural understanding, students only know how to apply a procedure to a problem that looks a specific way.

Conceptual Understanding

Across participants and tasks, there was little evidence that participants used a conceptual understanding to write a linear function from a graph or a table of values.

Rate of Change

Throughout all six mathematical task interviews, there was no sign of participants using an approach that would indicate a conceptual understanding of the rate of change. For instance, utilizing a covariation approach to find the rate of change would suggest that students have developed a conceptual understanding. Within the covariation approach, students identify that there is a relationship between two variables, which change together simultaneously. When using the covariation approach, students develop a global rule that shows how both variables change and how this rule applies to any coordinate pair on the linear function. However, participants within the current study did not show any instances of utilizing a covariation approach in either mathematical task.

In Task 2, participants had the opportunity to demonstrate a conceptual understanding of the rate of change through verbally explaining the meaning of the rate of change. Participants were encouraged to relate the rate of change back to the real-world problem. However, five of the six participants struggled to explain the rate of change. If students used a covariation approach, they might have mentioned the relationship between the amount spent, in dollars, it took Tanya to make any number of cards. In their attempt to explain the rate of change, only one participant referred to the relationship between the money Tanya spent for each card that she made. Within this study, it is unclear whether participants understood that a relationship existed between the quantities that comprised the rate of change. Because of their limited conceptual understanding, it might be difficult for students to apply their knowledge to make connections and solve real-world problems that involve linear functions outside of their mathematics classes.

Y-Intercept

Although participants recalled appropriate procedural approaches to find the y-intercept from both a graph and a table of values, their reliance on procedures may have limited their conceptual understanding of the y-intercept. In Task 2, participants were asked to describe the meaning of the y-intercept in relation to the context of the real-world problem. An accurate response about the meaning of the y-intercept would describe in some way that Tanya spent $4.50 on supplies to make greeting cards. Although Julie found that the y-intercept was $2.50 instead of $4.50, she was the closest out of the participants when describing the meaning of the y-intercept. Julie explained, "The 0 is in the left column. The number of cards. So, she spends $2.50 for 0 cards." The other participants shared incorrect responses, which included explanations about the total amount of money that Tanya spent or the amount of cards she had left. Based on their responses, participants did not demonstrate a conceptual understanding of the y-intercept, and they had a difficult time viewing the y-intercept as its own entity. Participants shared that the y-intercept was in some way related to the rate of change. One potential reason participants struggled to explain the y-intercept in terms of the context of a real-world problem was because of their dependence on procedural approaches and algebraic equations. Because participants followed a series of arithmetic steps to solve for the y-intercept, their work lacked any connection to the variables in the problem and their relationship to each other. As a result, participants found a numerical value that meant little to them in terms of the context of the problem.

Discussion

By conducting a mathematical task interview, students with LD were given the opportunity to share their understanding and the manner in which they approached tasks on linear functions that were represented in different ways. Because research on students with LD in mathematics, particularly in high school, is limited (Gersten et al., 2009; Watt et al., 2016), little was known about how they think about linear functions. To better support students with LD in mathematics, teachers may wish to recognize, honor, and integrate students' knowledge and misconceptions within their daily instruction. Not only do the findings from this study share the content knowledge of six students with LD on algebraic concepts, but also the findings contribute to the literature on students with LD who identify as Black or Latinx. In terms of higher-level mathematics courses and content such as algebra, the voices of these students with LD have too often been left out of the literature (Watt et al., 2016).

Overwhelmingly, participants in this study used procedural approaches to find the rate of change and y-intercept of a linear function in both a graphical representation and a table of values. Utilizing a procedural approach is a valid method that students can use to write a linear function. Based on the nature of procedural approaches, students are often taught and encouraged to memorize the steps of a procedure in a particular context. While students may successfully apply that procedure immediately after it has been taught, they may find it difficult to know when and how to apply that same procedure in the future or in different contexts (Gersten et al., 2009; Lambert, 2018). In their work, Adu-Gyamfi and Bosse (2014) highlighted that students often select their approach to solve problems with linear functions based on the task, and they found that students shared common, uncommon, and irregular rationales when explaining the approach they took to solve. Because linear functions can be represented in multiple ways, students may find it difficult to identify and apply an appropriate procedure for each of these different representations. The results of this study suggest that these challenges may also occur for students with LD who use a procedural approach.

When teaching linear functions, teachers may want to consider the ways in which teaching procedures in isolation can hinder students' problem-solving skills. Instead, teachers can encourage students to make connections between procedures and larger concepts so that students can apply procedures in tasks presented in a variety of ways. For example, in their work, Brenner et al. (1997) found that students who received instruction that incorporated multiple representations were more successful when applying their problem representation skills to new problems than students who did not receive this instruction. These students were more likely to use tables, diagrams, or equations to represent functions compared to students who did not receive the same curriculum. There is a need to extend Brenner et al.'s (1997) research to students with LD. In particular, studies on standards-based mathematics instruction that encourages the use of multiple representations and includes students with LD is imperative. When studying instruction, research can measure the performance of students with LD quantitatively. However, research should also qualitatively explore how students with LD perceive this type of instruction and the role this instruction plays in the development of their understanding of the mathematical content. Results from this research can inform instruction and curriculum design so that both general education and special education teachers can utilize these approaches to support students with LD in the development of their conceptual understanding.

Teaching students to rely on procedures may restrict students in their effort to justify their work and check their solution. In his work, Knuth (2000) found that students tend to rely on algebraic equations when solving problems with linear functions. Students may even disregard other representations that are more efficient to use. A reliance on certain procedural approaches and representations may lead students to make mistakes in either following the steps of a procedure or in their calculations. In the current study, several participants made errors when following the steps of the slope formula and using substitution with algebraic equations to find the y-intercept. In addition, participants within the present study made several calculation errors, even though they were permitted to use a calculator.

Because of the reliance on procedures, students may not acknowledge that they can transition between representations to justify their work or check their answer. For instance, only two participants that used procedural approaches in this study found the correct y-intercept for Task 1. Because participants were given a graphical representation in Task 1, they could have checked their work by graphing the y-intercept that they found. One participant in the current study found that the y-intercept in Task 1 was -44. If she attempted to graph a y-intercept of -44, she could have visually seen that this y-intercept would not fit the linear function. If this participant had gone back to the graph, she could have determined that her value for the y-intercept was incorrect. To encourage students to check their work and justify their solution, teachers may want to consider the ways in which they can teach students multiple procedures, show how procedures and concepts relate, and make connections between representations. Furthermore, teachers can create assignments that invite students to use at least two different representations to solve. By doing this, teachers are encouraging students to make connections between representations, but also giving students the choice of which two representations they want to utilize. To support students with LD in mathematics, further research is needed on the ways that students with LD apply procedures when problem solving. How accurate are students with LD in using the procedure to solve? What errors might they make? How do students with LD use their knowledge and understanding of the concept to check their work? In what ways do they justify their solutions? Interview studies that gather data on individual student thinking are essential so that mathematics education researchers and educators honor the voices, experiences, and needs of students with LD in mathematics.

Reliance on procedural approaches may exclude some students from accessing and understanding the content. When teaching procedural approaches to find the rate of change, teachers can utilize the slope formula, a pattern approach, or a triangle approach on a graphical representation, among other methods (Ellis, 2009; Zahner, 2015). Due to factors such as time constraints or previous experience, teachers may rely on teaching one method to find the rate of change over the others. However, all students learn differently, have different learning preferences, and varying learning needs. As a result, teaching and relying on one method to find the rate of change might exclude some students from learning the procedural method and accessing instruction. Because not all students learn the same, six high school students with LD that were interviewed about their mathematics instruction shared that they wanted their mathematics co-teachers to teach them multiple ways to solve the same problem, and they wanted their teachers to give them the opportunity to choose and apply whichever method worked best for them (Neill, 2021a). By only teaching one method to solve a mathematics problem, students with LD felt that their teachers were limiting their understanding (Neill, 2021a). To promote greater access and meet the learning needs of a diverse group of students, teachers may want to incorporate multiple methods to find the rate of change and y-intercept within their instruction. By doing so, teachers can make connections between differing procedural approaches, which might help students develop a more holistic and conceptual understanding of linear functions. For instance, teachers can encourage students to question why different procedures work and how they relate to each other. Additionally, the results of the current study suggest that the conceptual understanding of students with LD still needs to be further developed. To support students with LD, teachers can incorporate mathematical methods such as a covariation approach when teaching the rate of change (Ayalon et al., 2015). Another way teachers can support a conceptual development is by creating opportunities for students with LD to tie their knowledge of procedures to conceptual understanding so that students understand why procedures work.

In the present study, there were few, if any instances, in which participants demonstrated a conceptual understanding of linear functions. Because only mathematical task interviews were utilized in this study, it is unknown whether participants' teachers incorporated instructional practices and methods that aimed to develop students' conceptual understanding. Participants may have relied on procedural approaches because they felt more comfortable following a series of concrete steps. In a study of high school students with LD in mathematics, participants wanted their teachers to break down new mathematical content into steps and provide guided practice and repeated exposure (Neill, 2021a). The instructional preferences of these students with LD closely align with explicit instruction. During explicit instruction, teachers model or demonstrate how to solve a new mathematical concept and in doing so, break down that concept into series of steps (Doabler et al., 2012; Weibe Berry & Namsook, 2008) in an attempt to reduce the cognitive load for students (Archer & Hughes, 2011). When using this instructional practice, most mathematical concepts are taught to students as procedural approaches. Explicit instruction has been proven as an effective instructional practice for students with LD (Gersten et al., 2009; Graham & Harris, 2009; Hattie, 2009; Kroesbergen & Van Luit, 2003; Mastropieri et al., 1996; Swanson, 2001; Vaughn et al., 2000). Additionally, the Council for Exceptional Children and the Collaboration for Effective Educator, Development, Accountability, and Reform (CEEDAR; McLeskey et al., 2017) has identified it as a "High Leverage Practice" for students with disabilities. However, few studies have explored the use of explicit instruction when learning and applying procedures for more complex algebraic concepts such as linear functions (Gersten et al., 2009). Instead, many of the studies on explicit instruction have a narrow focus on a mathematical skill such as solving one-step addition and subtraction problems (Gersten et al., 2009). The current study sought to extend the research in this field by exploring students with

LD understanding of linear functions. Although participants could identify an appropriate procedure when working with linear functions, they struggled to do so accurately and consistently. Some participants in the current study could not completely apply the steps of procedures, and others made arithmetic errors leading them to incorrect solutions. Because qualitative research on students with LD is limited (Lambert & Tan, 2017), there is a need to increase the amount of research on students with LD that focuses on individual student thinking when applying procedures and exploring their conceptual understanding. More research in this field can help to identify students' strengths and weaknesses in utilizing procedural approaches. Findings from this research can help to inform instruction and the ways in which teachers may be able to make connections between procedural and conceptual knowledge so that teachers can meet students' needs and learning preferences.

Limitations and Implications

The purpose of this study was to explore how students with LD think about linear functions and the ways in which their understanding aligns or differs from the existing literature on students without disability labels. The results must be interpreted with caution, as the number of participants is limited. These qualitative findings cannot be generalized beyond these participants. However, findings could be transferred to other contexts that are similar. During the time that this study was conducted, the pandemic caused by COVID-19 forced school closures throughout the United States. As a result, participants received remote instruction for approximately three months of the 2019-2020 school year. Because the interviews were conducted in the summer of 2020, this experience may have influenced participants' understanding of the content and their overall disposition towards mathematics. Additionally, rather than being conducted in person, as previously planned, all interviews were conducted remotely through Zoom and PearDeak, a Google Slides Add-On. PearDeck allowed participants to complete the tasks within this interview on any electronic device. This platform may have also influenced participation, as students may have been more accustomed to completing their mathematics work on paper.

Although the design of this study has several limitations, the data gathered through the use of mathematical task interviews highlights the value of giving voice to students with LD. Throughout their mathematical task interviews, participants shared their thinking while solving mathematical tasks. The researcher gathered detailed information on participants' understanding of mathematical content, in particular how they approached the mathematical task and why. In addition to seeing their work, a mathematical task interview allows the interviewer to ask questions in the moment. After giving a mathematical task, teachers can only view the work students wrote on paper and their final answer. While teachers can follow students' work and measure the accuracy of the solution, teachers may not gain a deeper sense of students' understanding, especially if little work is written or the work is inaccurate or irrelevant. As such, mathematical task interviews are a valid and useful method of assessment and source of data that can be used to inform instruction. Teachers may want to consider incorporating mathematical task interviews within their instruction to learn more about students' thinking and learning. Results from mathematical task interviews can be used to identify additional areas of support for students. Furthermore, data gained from mathematical task interviews can be used to design instruction that better meets students' needs and write IEP goals for students with LD.

Further research is needed on instructional practices that seek to develop students with LD conceptual understanding of linear functions. In the current study, participants approached tasks on linear functions with a procedural understanding. Participants' procedural understanding may be attributed to their history of mathematics instruction. For instance, participants' reliance on procedures may be due to teachers spending more time teaching approaches such as the slope formula rather than a manifestation of their disability. Furthermore, in this study, participants' knowledge of linear functions, use of procedural approaches, and reliance on algebraic equations did not differ drastically from existing research and literature on students without disability labels. As such, additional research is needed on how teachers incorporate instruction that encourages students with LD to develop a deeper understanding of linear functions, such as utilizing the covariation approach to find the rate of change. Additionally, research on instruction that urges students with LD to make connections and transition between representations of linear functions is needed. Research in this area can help teachers plan and execute lessons that incorporate instructional practices that develop conceptual understanding while simultaneously addressing the strengths and misconceptions of students with LD. By considering students with LD thinking about linear functions, teachers can better support students with LD in mathematics.

CHAPTER IV

TEACHING RATE OF CHANGE TO STUDENTS WITH LD RESEARCH BRIEF

Linear functions is a concept that is typically introduced in middle school, and it continues to appear throughout higher levels of mathematics. To succeed in higher levels of mathematics, students need to build a strong foundational understanding of linear functions (Capraro & Joffrion, 2006; Dubinsky, 1993). However, students with and without disability labels may not develop a conceptual understanding of linear functions because of the level of abstract thinking that it entails (Brenner et al., 1997; Kieran, 1992). Thus, teachers play an important role in deepening students' knowledge of linear functions (Capraro & Joffrion, 2006; Zahner, 2015). Both real-world examples and multiple representations of the same linear function can assist students in their attempt to understand and make connections between their experiences and mathematical representations (Brenner et al., 1997). However, real-world tasks and tasks that incorporate multiple representations must be planned and implemented strategically to ensure that students make appropriate connections.

Research is limited on the perceptions that students with a learning disability (LD) have about their mathematics instruction and their understanding of mathematics content, in particular, algebraic concepts. To meet their needs and learning preferences, it is essential to understand how students with LD prefer to be taught in mathematics. In a study of six high school students with LD, students favored explicit instruction with multiple opportunities for guided practice and repeated exposure (Neill, 2021a). Students described that they preferred when their teachers showed them multiple ways to solve the same problem and provided them with opportunities for group work. During group work, students wanted to share their understanding with their peers and receive feedback and small group support from their teachers (Neill, 2021a). Additionally, students with LD felt that their mathematics instruction was rushed (Neill, 2021a).

In addition to exploring their instructional preferences, there is a need to know the ways in which students with LD approach tasks on linear functions. Six high school students with LD participated in a mathematical task interview study in which they completed a series of tasks on linear functions while sharing their thinking with the interviewer (Neill, 2021b). During their interview, students demonstrated a procedural understanding of linear functions. Students relied on procedures to find the rate of change and y-intercept. While five of the six students recalled an appropriate procedure to utilize, their attempts to accurately and consistently apply procedures when solving varied (Neill, 2021b). Teaching rate of change to students with LD should include evenly paced, multiply represented instruction that incorporates checks for understanding, opportunities for feedback, and inclusion of group work. Furthermore, instruction on the rate of change should shift from focusing on procedures to a conceptual understanding by encouraging the use of multiple representations, incorporating real-world problems, and focusing on the relationship between quantities. The purpose of this research brief is to address how students with and without disability labels approach tasks on linear functions, particularly finding the rate of change, and how instruction can be adjusted to promote students' development of a conceptual understanding.

Defining the Rate of Change and Y-Intercept

The rate of change and the y-intercept are two essential concepts of linear functions. Both the rate of change (Herbert & Pierce, 2012; Teuscher & Reys, 2010; Wilkie & Ayalon, 2018) and the y-intercept (Davis, 2007; Knuth, 2000) are often difficult ideas for students to understand conceptually. Ayalon et al. (2015) described the rate of change as a relationship between two variables in which a change in one variable is expressed based on a change in the other variable. In the early years of algebra, the rate of change may be presented to students as the slope or steepness of a line. Teachers and students often use the terms slope, rate of change, and steepness interchangeably (Knuth, 2000; Teuscher & Reys, 2010). Most textbooks refer to the y-intercept as the value of the y-coordinate when its x-coordinate is zero or the value of the y-coordinate when the linear function crosses the y-axis on a graphical representation (Knuth, 2000).

Common Instructional Practices Used to Teach the Rate of Change

Before being introduced to the rate of change, students receive instruction on related concepts such as the constant of proportionality. Similar to the rate of change, the constant of proportionality can be found from a table of values, a graphical representation, and a real-world description. Additionally, students can write an algebraic equation to represent the constant of proportionality. A strong awareness of the constant of proportionality can assist students as they explore and make meaning of the rate of change. Although students receive repeated exposure to the rate of change throughout their mathematics education, they experience difficulty demonstrating a conceptual understanding of this concept.

Because of the abstract nature of linear functions and various external pressures, teachers may spend more time focusing on the use of a procedural approach to find the rate of change of a linear function (Teuscher & Reys, 2010; Zahner, 2015). Procedural knowledge refers to students' understanding of mathematical language, symbols, rules, and algorithms. In this case, students follow step-by-step procedures to solve a problem on a particular skill (Capraro & Joffrion, 2006). Usually, students complete these procedures without understanding the reasons behind why the procedures work in aiding them to solve the task at hand (Capraro & Joffrion, 2006; Stump, 2001). Using the slope formula is one common procedural approach that students use to find the rate of change of a linear function (Teuscher & Reys, 2010). In this case, students select two ordered pairs from the linear function. Then, students use these ordered pairs to find the difference between the *y* coordinates and the difference between the *x* coordinates. Another procedure that students tend to use is creating a right triangle between two ordered pairs on a graphical representation in which students find the length of each leg of the triangle by counting the units on the graph. Students that use the triangle procedural approach often identify that the rate of change of the function is a fraction, such as $\frac{4}{1}$, rather than writing it as the integer of 4 (Zahner, 2015). Students that write the rate of change as a fraction show confusion about the role the quantities of the numerator and the denominator play in making up the rate of change (Herbert & Pierce, 2012).

Other teachers encourage students to find patterns within number tables in their effort to determine the rate of change (Ellis, 2009). Students who use this approach will concentrate on finding patterns between the *x* coordinates and the *y* coordinates separately, rather than identifying a coordinate pair as one entity. Similarly, the correspondence approach is an approach in which students develop a rule or pattern that allows students to solve for any value of *y* based on the value of *x*, known as the input-output method. In this case, students may substitute any given value of *x* into the linear function to solve for the value of *y* (Wilkie & Ayalon, 2018). While students have shown some level of success in using these procedures to find the rate of change (Ayalon et al., 2015; Wilkie & Ayalon, 2018; Zahner, 2015), instruction that solely emphasizes a procedural approach limits students in their attempt to understand the rate of change at a deeper level (Capraro & Joffrion, 2006; Herbert & Pierce, 2012; Stump, 2001; Teuscher & Reys, 2010).

How Does Instruction Influence Student Understanding?

Across the literature, there has been an emphasis on the importance of developing a conceptual understanding of linear functions (Capraro & Joffrion, 2006; Ellis, 2007; Kieran, 1992). However, because instruction often focuses on procedural approaches, many students still show only a procedural understanding of the rate of change (Ellis, 2009; Teuscher & Reys, 2010; Zahner, 2015). Due to their instruction and procedural understanding, students often struggle to explain the meaning of the rate of change, develop incomplete understandings, and show a reliance on algebraic equations.

Difficulties Explaining the Rate of Change

While students can find the rate of change accurately using their preferred procedure, students often struggle explaining the meaning of the concept. Students may describe the rate of change as the "rise over run" or the "up and over" (Zahner, 2015). Because some students cannot distinguish the difference between the terms slope, rate of change, and steepness, they use all of these terms in a similar manner (Teuscher & Reys, 2010). In her work, Stump (2001) found that pre-service teachers were surprised that their students could not describe the numerical value of the rate of change. Additionally, students could not explain the meaning of the rate of change in terms of the context of the problem. When teachers focus their instruction on encouraging students to find patterns in tables, students experience issues developing accurate global rules and explaining the relationship between the independent and dependent variables (Ellis, 2009). Instead, students make comments about the variables separately, such as "on the *x* side, it's going up by ones, and on the other side, it's going up by sevens" (Ellis, 2009, p. 485). Similarly, in a study of six high school students with LD, only one student could explain the meaning of the rate of change based on a real-world problem (Neill, 2021b). As a result of using a procedural approach, students show a lack of knowledge about the relationship between the two variables that make up the rate of change. Furthermore, when taught to use procedures primarily, students grapple with applying procedures and methods when solving, particularly when given tasks in different representations (Capraro & Joffrion, 2006).

Developing Incomplete Understandings

During their education, students develop incomplete understandings about the rate of change. For example, students struggle to differentiate between the terms slope, rate of change, and steepness (Stump, 2001). Teuscher and Reys (2010) explained that the steepness of a line refers to the visual perception of the graph of a linear function. However, the slope and rate of change of a linear function refer to the relationship between the independent and dependent variables. Possibly due to their limited conceptual understanding, not only do students have a difficult time explaining the difference between steepness, slope, and rate of change, but they also cannot identify the relationship among these concepts (Teuscher & Reys, 2010). To help support students' development, teachers can introduce steepness using real-world examples such as the roof of a house or a building. Through exploring the slope of various roofs, students can deduce that a larger number corresponds with a steeper slope (Teuscher & Reys, 2010).

While students are able to identify the rate of change from a given task, they often disregard the sign of the rate of change. In their work, Teuscher and Reys (2010) found that students did not acknowledge that a rate of change of 2 was different from a rate of change of -2. To address this misconception, teachers should emphasize that the sign of the rate of change provides vital information about the relationship between the independent and dependent variables, such as a rate of change of -2 indicates that as the independent variable increases, the dependent variable decreases. Because of their incomplete understanding regarding the important role that the sign of the rate of change plays in a linear function, students may not be acknowledging the relationship between the two variables. Table 7 provides educators with strategies to address potential misconceptions or misunderstandings students have about the rate of change.

Table 7

Addressing Students' Misconceptions and Misunderstandings

Common Misconceptions and/or Misunderstandings	Research-Based Interventions
Students rely only on a procedural approach.	Encourage the use of a covariation approach. Students can view and understand that the rate of change is two quantities that change together.
Students see the x-coordinates and y-coordinates as separate entities.	Include problems on speed to allow students to identify a relationship between the two different variables in terms of distance and time.
Students view the rate of change only as one quantity instead of a relationship between two quantities.	Incorporate situational problems with different variables, such as the speed of an elevator building.
Students struggle to differentiate between steepness, slope, and rate of change.	Use real-world examples, such as the roof of a house or mountain. Give students several examples in which they can deduce that a larger rate of change corresponds with a steeper slope.
Students disregard the sign of the rate of change (positive versus negative).	Show students graphical representations of a rate of change that is positive and negative. Emphasize the direction as well as the relationship between the independent and dependent variables.
Students rely on one representation, typically an algebraic representation.	Design and implement instruction that incorporates multiple representations, which allows students to make connections between representations.
Students lack understanding about the Cartesian Connection.	Encourage students to link coordinate pairs from a table to a graph. Draw students' attention to the connection between coordinate points on a graph and a table, and how both of these representations connect to the algebraic equation of the linear function.

Relying on Algebraic Equations

When it comes to working with linear functions, students prefer to use an algebraic representation (Knuth, 2000). Students may rely on algebraic equations because, typically, students are introduced to linear functions through equations in slope-intercept form (Knuth, 2000). For example, in his study of 178 high school students, Knuth (2000) found that students relied on using an algebraic equation even though using a graphical representation would have been easier and more efficient. In fact, Knuth noted that the task in his study was created in an effort to compel students to use a graphical approach. Furthermore, in Knuth's work, students relied on an algebraic approach so much so that they did not even acknowledge that a graphical representation could be used to solve the task or justify their answer. One reason students may disregard using a graphical approach is that they struggle in their attempt to make connections between ordered pairs on a graph and algebraic equations (Hart, 1981; Moschkovich et al., 1993). When students lack the skills needed to make associations between representations, they show a limited understanding of linear functions. Depending on the use of only one representation of a linear function can be problematic for students because they will struggle to solve problems presented in various ways, some of which may be unfamiliar to them. As such, it is important that students learn to make connections between algebraic equations and graphical representations. With this knowledge, students can check their work and justify their solution.

Instructional Practices to Help Students Develop a Conceptual Understanding

Researchers have suggested that teachers should not ignore the development of procedural knowledge completely. Procedural and conceptual understanding are not separate entities; rather, both are needed for student success. As such, a hearty mix of procedural understanding and conceptual knowledge is necessary for students to form a more complete picture of linear functions (Adu-Gyamfi & Bosse, 2014; Capraro & Joffrion, 2006; Knuth, 2000). Conceptual understanding refers to students linking new ideas to their previous conceptions and building knowledge that is rich in relationships (Stump, 2001). In their work, Capraro and Joffrion (2006) shared that "without conceptual understanding, procedures mean almost nothing. Connections make mathematics meaningful, memorable, and powerful" (p. 163).

Teaching Quantities to Develop Conceptual Understanding

A solid conceptual understanding of a mathematical concept increases students' problem-solving skills, as it encourages students to approach tasks in any manner they deem fit. Furthermore, students can justify their solution based on their knowledge and its relationship to the problem. When students build their conceptual understanding, they possess the logic and reasoning as to why various procedures can help to solve the task. However, with just a procedural understanding, students may only know how to apply a procedure to a specific type of problem that looks a certain way.

To support students' development of a more conceptual understanding of the rate of change, teachers should allocate both time and experience for students to transition from a procedural conception to a structural conception. One way to do this is by encouraging students to focus on quantities and how those quantities relate to each other. In a study of a class of seventh-grade students, the teacher sought to develop students' quantitative reasoning and understanding of the rate of change by incorporating examples on speed and gear ratios (Ellis, 2009). Rather than viewing the variables as separate entities, students demonstrated their understanding that a relationship between the two variables existed. For example, students identified a relationship between the variables in terms of the distance, in feet, and the time, in seconds. More importantly, students were better able to extend their understanding and make sense of problems that differed from examples previously given. For instance, students were able to reason and develop accurate global rules about linear functions in slope-intercept ($y = mx + b$) form. However, students in another class that focused only on patterns in a table could not justify examples in slope-intercept form. Rather, students in this class shared that equations in the form of $y = mx + b$ were not linear because the pattern that they developed did not fit all of the data points on the given table (Ellis, 2009).

The Covariation Approach to Develop Conceptual Understanding

Teachers can also introduce the rate of change through a covariation approach. A covariation approach "involves analyzing, manipulating, and comprehending the relationship between changing quantities" (Ayalon et al., 2015, p. 323). The covariation approach highlights that the rate of change is not one quantity or the other, but rather a new entity that is comprised of changes within both variables. Teachers can incorporate situational problems with different variables that students can manipulate and relate to, such as the speed of an elevator in a building. Students can explore the relationship between the speed, in seconds, it takes an elevator to get to various floors of a building (Ayalon et al., 2015). Using the covariation approach, students develop a global rule that shows how both variables change and how this rule applies to any floor in the building. Verbal explanations about the rate of change by students using this approach would include descriptions about both variables, such as the rate of the elevator is three floors per second (Ayalon et al., 2015). While studying the use of the correspondence and covariation approaches, in their work, Ayalon et al. (2015) found that students tend to rely on the correspondence approach. However, students that used a covariation approach were more successful in developing accurate rules to represent functional relationships and completing their given tasks. These results may be because a covariation approach focuses on the relationship between the quantities, whereas the correspondence approach is more procedural. Rather than introducing the rate of change as a calculation to determine a single value, teachers may want to introduce and emphasize that the rate of change is a relationship between the changes in two quantities (Herbert & Pierce, 2012).

Teaching Students to Make Connections between Representations

Linear functions can be represented in multiple ways. A linear function can be written as an equation in slope-intercept form ($y = mx + b$) or standard form ($ax + by = c$), and it can be seen in the form of a table, graph, or a real-world description. With a strong conceptual understanding, students can recognize that the same linear function can be represented in these various ways, and students can move back and forth between these representations. While studying classroom instruction designed to emphasize and incorporate multiple representations, Brenner et al. (1997) found that students who received this type of instruction were more successful when applying their problem representation skills to new problems. They were more likely to use tables, diagrams, or equations to represent functions than students who did not receive the same curriculum. Instead, students that did not receive instruction on multiple representations of linear functions relied on rote memorization of symbol manipulation (Brenner et al., 1997). By designing a unit that emphasizes problem representation skills, teachers give students the opportunity to make connections between multiple representations and deepen their understanding.

A conceptual understanding of linear functions is one that is rich in relationships. Students cannot understand the various representations of linear functions in isolation (Wilkie & Ayalon, 2018). Instead, students should see and understand how each representation is related to one another. The Cartesian Connection plays a role in helping students to make connections between representations. Wilkie and Ayalon (2018) shared that

> The Cartesian Connection is considered a critical translation for students to learn to make between algebraic equations and their graphs. It is also foundational for relating the rate of change in a linear equation (described by the coefficient of *x*) with the gradient (slope) of its graph. (p. 504)

Often, students are exposed to the Cartesian Connection early in their algebraic educational experience. Because teachers are under the impression that the concept was previously taught, the topic is rarely revisited (Knuth, 2000). Additionally, it is often assumed that not only do students comprehend the concept, but they also retain knowledge over time (Knuth, 2000; Knuth et al., 2005). However, even students enrolled in higher-level math classes such as Advanced Placement (AP) calculus, have shown difficulty in accurately recalling information about the rate of change that was taught in years prior (Teuscher & Reys, 2010). As such, it is imperative that teachers check students' understanding of the Cartesian Connection and revisit the concept, if necessary. When discussing the rate of change, teachers can emphasize the Cartesian Connection and highlight to students that any point on the graphical representation of the linear function is a solution to the algebraic equation (Knuth, 2000). Paying particular attention to a graphical representation during instruction is essential because students often struggle to make connections between an equation and a graph (Wilkie & Ayalon, 2018). Because students tend to feel more comfortable with a table representation than a graph (Wilkie & Ayalon, 2018), teachers can draw students' attention to the connection between ordered pairs on a graph, a table, and an algebraic equation. More importantly, encouraging students to make connections between representations will help students take and use knowledge developed in one context and apply it to other contexts (Wilkie & Ayalon, 2018).

Strategically Incorporating Real-World Problems

Incorporating real-world problems can help students establish their conceptual understanding of the rate of change. It can be difficult for students to "see" that the slope of the line is its rate of change because it is a relationship between two variables (Lobato et al., 2003; Noble et al., 2004). By including real-world problems, students can use their everyday experiences in an attempt to better understand how the independent and dependent variables change together to create the rate of change (Leinhardt et al., 1990). For example, students can find success and enhance their performance on tasks with linear functions when time-based graphs are included within instruction (Leinhardt et al., 1990). In addition, real-world representations such as roofs, mountains, and wheelchair ramps can help students understand steepness, compare different values of steepness, and differentiate between steepness and slope (Stump, 2001). However, teachers should not assume that just because they include real-world examples within their instruction that students will automatically understand the rate of change (Stump, 2001). Real-world examples must be meaningful to students and allow students to draw on their prior knowledge. Additionally, while working with real-world problems, students may make incorrect connections or attempt to make connections where they do not exist based on their prior experiences (Davis, 2007). With appropriate instruction, real-world problems can be used as a tool for students to comprehend the relationship between the changes in two variables.

Supporting Students with Learning Disabilities

Federal legislation, such as the Individuals with Disabilities Education Act (IDEA; 2004) and No Child Left Behind (NCLB; 2002), require that schools educate students with LD with their peers without disability labels and provide students with LD access to the general education curriculum. As a result, there has been an increase in the number of students with LD educated in a general education setting for 80% or more of their school day (Cortiella & Horowitz, 2014). Often, schools in the United States place students with LD in an Integrated Co-Taught (ICT) class in which a subject-area or grade-level teacher and a special education teacher work together as co-teachers to provide instruction to general and special education students. Both co-teachers are responsible for implementing the standards-based curriculum to all students, including students with LD.

In 2014, the National Council of Teachers of Mathematics (NCTM) published the *Principles to Action: Ensuring Mathematical Success for All* calling for teachers to encourage students to be an active part of the learning process and construct their own knowledge. The notion was that through discussion, exploration, and inquiry, students would develop a deeper conceptual understanding. However, inquiry-based teaching practices often conflict with many of the research-based practices identified as beneficial for students with LD. Because these pedagogies differ, questions, and possibly tensions may arise between co-teachers as they attempt to navigate the best way to ensure that students with LD have the appropriate access to the standards-based curriculum. Rather than teachers utilizing one instructional practice or another, co-teachers can work together to incorporate various instructional practices to support all students. For instance, while implementing the Five Practices for Orchestrating Productive Mathematics Discourse (Stein et al., 2015) teachers can include instances of explicit instruction, which has been proven as useful for students with LD (Gersten et al., 2009; Graham & Harris, 2009; Kroesbergen & Van Luit, 2003; Mastropieri et al., 1996; Swanson, 2001; Vaughn et al., 2000). The following section will highlight the ways in which teachers can use explicit instruction, student verbalizations, and heuristics to help students develop their understanding of the rate of change. While these practices have been found advantageous for students with LD in mathematics, teachers can use other instructional practices to assist students with LD and are not limited to only these three.

Explicit Instruction

Incorporating instances of explicit instruction can support students with LD as they develop their understanding of the rate of change. Within explicit instruction, teachers model and break down a mathematical concept into steps (Doabler et al., 2012; Weibe Berry & Namsook, 2008). By breaking down concepts into discrete parts, teachers help reduce the cognitive load placed on students based on their current skills (Archer & Hughes, 2011). Following the completion of the demonstration, teachers lead students in guided practice. During this time, teachers ask questions and elicit participation from students while monitoring their responses and providing feedback (Doabler & Fien, 2013; Hughes et al., 2017). Teachers can use students' responses to adjust instruction in an effort to meet the needs of students. In addition to being an effective research based practice for students with LD (Graham & Harris, 2009; Kroesbergen & Van Luit, 2003; Mastropieri et al., 1996; Swanson, 2001; Vaughn et al., 2000), high school students with LD shared that they preferred when their mathematics teachers modeled and broke down content into steps (Neill, 2021a).

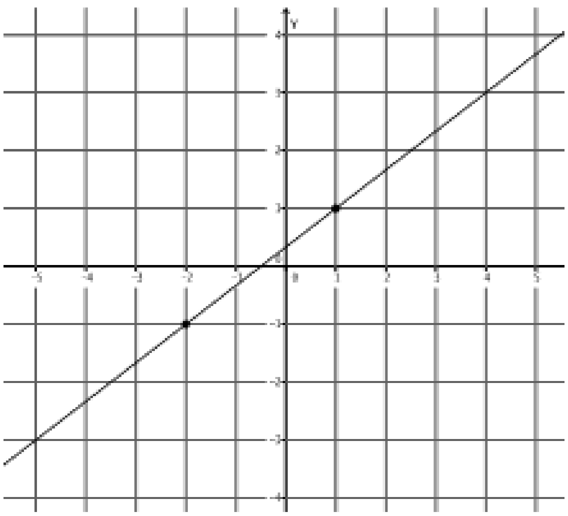
By using explicit instruction, teachers can demonstrate to students that the sign of the rate of change is meaningful. More specifically, using several graphical representations can help teachers show students the difference between a positive rate of change and a negative rate of change. Figure 3 shows four different linear functions, three of which have a positive rate of change and one which has a negative rate of change.

Figure 3

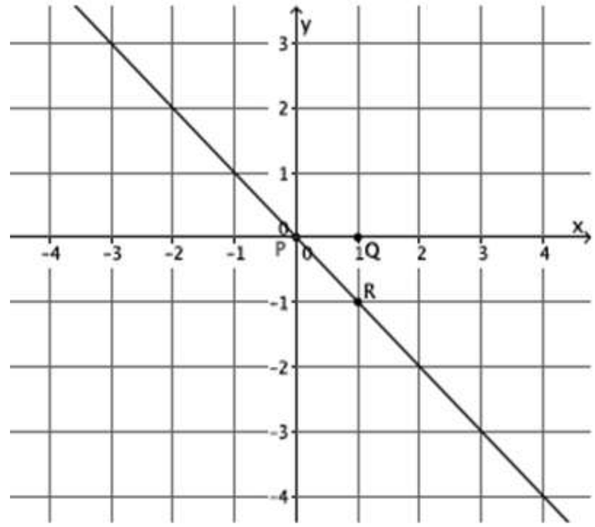
Graphical Representation of Positive versus Negative Rate of Change

Linear Functions	

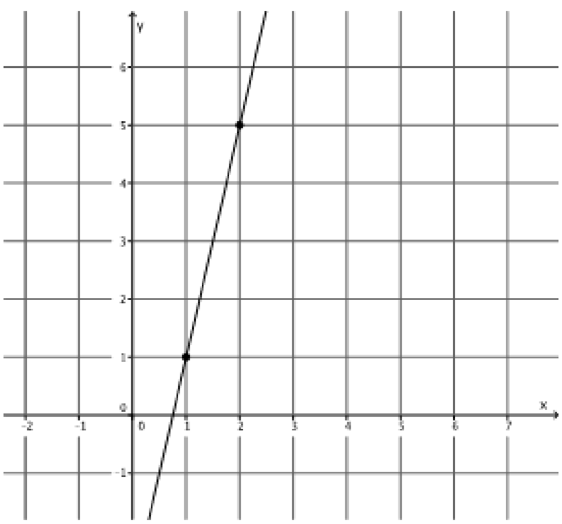
Function A



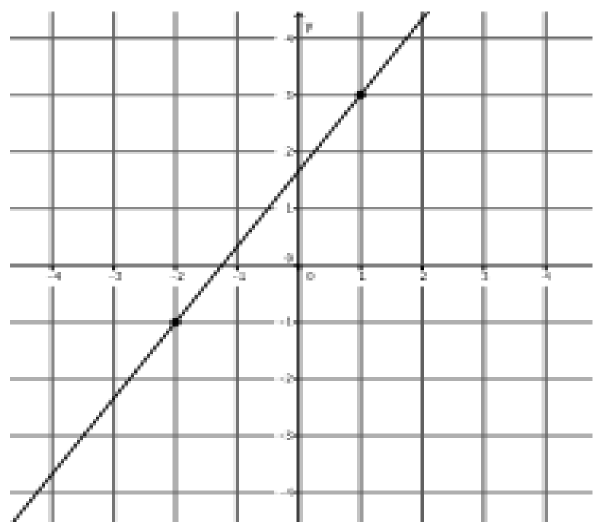
Function B



Function C



Function D



Teachers can present the four graphs shown in Figure 3 to students. Then, teachers invite students to share any similarities or differences that they notice between the graphs. Through explicit instruction, teachers emphasize to students that Function B looks different from Functions A, C, and D. While Functions A, C, and D have a positive rate of change, Function B has a negative rate of change. Through explicit instruction, teachers call attention to visual cues on graphical representations that can assist students in understanding the importance of the sign of the rate of change.

During explicit instruction, teachers can use a think-a-loud to model for students how to interpret the relationship between the variables using a description. For instance, teachers can show students a graphical representation of the linear function $y = 50x + 100$, where x is the number of hours worked, and y is the total amount of money earned. Using this example, teachers can model how to read the graph and explain that as the number of hours worked increases, the total amount of money earned also increases. Through demonstrations and guided practice, students develop the skills to explain the relationship between the variables. When faced with a graphical representation in the future, students use their instruction on interpreting the relationship between the variables to make a connection between their description and the value of the rate of change. Explicit instruction does not need to span for an entire class period. Rather, explicit instruction can occur at the start of the lesson to provide all students access to the aligned task, while students are working in small groups, or at the end of the lesson when teachers are making connections between students' work.

Explicit instruction also allows for repeated exposure to a concept. Repeated exposure provides students with and without disability labels the chance to see the mathematical concept multiple times and gives them several opportunities to develop and practice their skills. High school students with LD shared that they wanted their mathematics teachers to show them how to solve a problem multiple times (Neill, 2021a). Additionally, they wanted their teachers to provide them with several opportunities to practice their new skills under the guidance of their teachers (Neill, 2021a). Using explicit instruction and repeated exposure can help students make connections between multiple representations of the same linear function. During instruction, teachers model for students how to identify different representations of a function with the same rate of change. Even though a graphical representation, a table of values, and an algebraic equation may look different, when graphed, all three representations would look the same. Furthermore, teachers take the time to demonstrate to students this process and post the various representations side-by-side for students to view and compare. To allow for repeated exposure and practice, teachers give students open-ended tasks that encourage them to use more than one representation to solve the task and justify their solution.

While explicit instruction and repeated exposure are important practices that support students with LD, both practices should not be the sole source of instruction for students with LD (Geary et al., 2008). Students with LD are capable of constructing their own knowledge based on their previous understandings. As such, students with LD deserve access to a standards-based curriculum, which includes participating in inquiry-based instruction (Lambert, 2018). Teachers can use explicit instruction to support students' understanding at various points of the lesson while still encouraging them to make meaning of the larger task through inquiry and discussion with their peers.

Student Verbalizations

Student verbalizations is another instructional practice that has proven to be effective for students with LD in mathematics (Gersten et al., 2009). Student verbalizations encourage students to explain their thinking process aloud. By prompting students to share their thinking, teachers can scaffold instruction, evaluate students' problem-solving strategies, and recommend that students discuss their mathematical approaches with their peers or their teachers (Gersten et al., 2009). Teachers can prompt students to share their thinking with a peer, such as a think-pair-share or in a small group. Additionally, teachers can give students with LD self-questioning scripts while they are working on the given task. These self-questioning scripts can include general questions such as “Have I read and understood the task?” and “Are there any words whose meaning I have to ask?” (Hutchinson, 1993). Because the rate of change is a more abstract algebraic concept, self-questioning scripts can also include questions that are specific to the mathematical concept and/or the task that students are working on that day. In regard to the rate of change, script questions may include, but are not limited to the following: (a) What representation(s) can I use to solve this task?, (b) What are the variables being given/shown?, (c) Is the rate of change positive or negative? How do I know?, and (d) How are the variables changing? What is their relationship to each other? While self-questioning scripts are an effective strategy for students with LD, teachers may want to show students how to use self-questioning scripts. Simply giving students a self-questioning script will not be effective. Without modeling or demonstrating how to use the script, students may become more confused, frustrated, and overwhelmed. Teachers can use explicit instruction to model for students how to use self-questioning scripts. After practicing a few times with their teachers, students will begin to feel comfortable using the self-questioning scripts independently. Then, students can be given a self-questioning script to assist them during an inquiry-based task.

Heuristics

Teachers can also use heuristics to support students with LD in developing their understanding of the rate of change. Similar to self-questioning scripts, a heuristic is a tool that acts as a self-regulation strategy for students in their effort to solve a mathematical task (Maccini

& Hughes, 2000). When using a heuristic, students follow a series of steps to help them identify key information and develop a course of action to complete the mathematics task (Watt et al., 2016). Including heuristics within mathematics instruction gives students with LD a tool to organize, process information, and self-regulate their work (Watt et al., 2016). One heuristic that students can use is the problem-solving guide, which includes the following four steps: (1) read it and make it simple, (2) get a strategy, (3) work the problem, and (4) check it (Woodward et al., 2001). Within this guide, teachers can include several questions and prompts for students to follow and provide suggestions for problem-solving strategies such as drawing it, making a table, looking for a pattern, or working backward (Woodward et al., 2001). While the four steps should remain consistent so that students can apply this problem-solving guide to a variety of questions, the suggestions for problem-solving strategies can be tailored to the given task or concept. In regard to the rate of change, strategies and prompts can include, but are not limited to, sketch a graph, create a table, describe the way that the graph looks, tell your friend how the variables are changing, and use two representations to prove your answer. As with the self-questioning script, students with LD should be explicitly taught how to use the problem-solving guide so that there is no confusion or frustration. Students should view self-questioning scripts as a tool to aid their work rather than viewing the script as more work for them to complete.

Conclusion

When teaching the rate of change, educators may want to consider moving from instruction that emphasizes the use of procedural approaches to instruction that seeks to develop students' conceptual understanding. Although students receive repeated exposure to linear functions and the rate of change throughout their experiences in algebra, many students do not show a conceptual understanding of the rate of change. Because of this, students cannot explain the meaning of the rate change, they struggle to differentiate between a positive and negative rate of change, and they tend to rely on an algebraic approach when working on tasks that involve linear functions. To support students, teachers can include instruction that focuses on quantities with real-world connections such as speed. In doing so, teachers give students the opportunity to better understand the rate of change as a relationship between two quantities. Furthermore, teachers can design instruction to highlight the connections between different representations of linear functions. Even though incorporating multiple representations and real-world problems into daily instruction can support students' development, teachers may want to consider providing appropriate instruction that makes a clear connection between representations so that students do not make inaccurate connections or develop misconceptions. The possibility that these incomplete understandings and misconceptions can stay with students for years is even more problematic (Teuscher & Reys, 2010). With the notion that algebra and algebraic concepts act as a gatekeeper in mathematics (Capraro & Joffrion, 2006), it is important that teachers prepare students for future success in higher-level mathematics courses by helping students form a deep and conceptual understanding of the rate of change.

CHAPTER V

CONCLUSION

This study was designed based on the researcher's belief that to support students with a learning disability (LD) in the classroom, teachers and mathematics education researchers must give voice to students' needs, preferences, and knowledge. In addition, educators must honor the voices of students with LD when planning curriculum and instruction. Because research on high school students with LD in mathematics is limited (Gersten et al., 2009; Lambert & Sugita, 2016), it is somewhat unknown whether or not mathematics co-teachers of Integrated Co-Taught (ICT) classes are meeting the needs of their students. Furthermore, if little research on students with LD in high school mathematics exists (Lambert & Sugita, 2016; Watt et al., 2016), one must wonder which research co-teachers are using when making instructional decisions. While teachers could plan instruction based on information presented in professional development, teachers may want to consider that students with LD are experts in their classroom experience. As such, their voices should not only be included in the literature but also incorporated into daily classroom instruction.

In an effort to give voice to students with LD, qualitative methods were used in this study. Chapter II discusses the results of semi-structured interviews with high school students with LD. Interview questions sought to learn more about the instructional practices used in ICT mathematics classes and participants' perceptions of those practices. Findings suggest that participants felt explicit instruction with guided practice was advantageous for their learning. While participants did not use the term explicit instruction, they referred to practices used within explicit instruction, such as teachers modeling and breaking down new mathematical concepts into clear and unambiguous steps (Archer & Hughes, 2011; Doabler et al., 2012; Weibe Berry & Namsook, 2008). Participants preferred when their teachers demonstrated how to solve a problem using a series of steps, and they wanted to practice these steps under the guidance of their teacher several times before completing their work independently. Not only did participants like when their teachers showed them how to solve a problem or concept in multiple ways, but also when their teachers allowed them to use the mathematical method that worked best for them. Additionally, participants discussed that the pace of instruction was often too quick for them, and they appreciated it when teachers gave them the opportunity to work in a group with their peers. Based on their history of instruction in mathematics, participants may have only been exposed to explicit instruction, and as such, prefer it. Because little is known about students with LD participation in a standards-based curriculum (Lambert & Sugita, 2016), further research is needed on how students with LD perceive other types of instruction, such as inquiry-based instruction.

In Chapter III, the researcher used a mathematical task interview, also known as a clinical interview in mathematics research education (Clement, 2000), to learn more about students with LD understanding of linear functions. Linear functions is a concept that spans across grades. Because linear functions is a topic that appears throughout algebra and builds the foundation for concepts taught in calculus (Capraro & Joffrion, 2006; Dubinsky, 1993), there is a need to know how students with LD think about this concept. Findings from this study suggest that participants demonstrated a procedural understanding in which they utilized procedural approaches such as the slope formula and substitution when determining the rate of change and y-intercept. While five of the six participants identified appropriate procedural approaches to find the rate of change and y-intercept, their accuracy and consistency applying procedures varied across tasks that were represented in different ways. Teachers may want to incorporate and encourage the use of multiple representations and real-world problems to support students with LD as they develop their conceptual understanding.

Chapter IV is a potential resource for teachers, both general and special education, as they plan instruction for students with and without disability labels on linear functions. The article highlights that students often possess a superficial understanding of linear functions because they rely on procedural approaches. However, the article explains the importance of encouraging students to develop a conceptual understanding of linear functions and the role that teachers play in planning instruction that supports a more in-depth understanding. Furthermore, the article provides several research-based strategies for teachers to incorporate into their instruction, such as real-world problems and multiple representations.

Bridging Gaps in Research and Pedagogy

The ways that researchers study students with LD may limit teachers' efforts to educate students in the classroom appropriately. In terms of the field of mathematics education, Lambert and Tan (2017) argued that students with LD and their voices have been excluded from research because they are framed as "problematic." The research that does exist on students with disabilities is mostly quantitative in nature (Gersten et al., 2009; Lambert & Tan, 2017; Watt et al., 2016). As a result, in regard to students with disabilities, little analysis of student thinking exists. Rather, students with disabilities are studied using aggregate test scores (Lambert & Tan, 2017). Furthermore, there are stark differences between the ways in which students with and without disability labels are studied in mathematics. Lambert and Tan (2017) found that only 6% of research studies on students with disabilities were qualitative compared to 50% of research studies on students without disabilities. Additionally, 86% of research studies on students with disabilities were quantitative, whereas only 35% of research on students without disabilities were quantitative. In an attempt to begin to fill this very prevalent and problematic research gap, qualitative methods were used to honor the voices of students with LD, their preferences, and their knowledge. Through interview studies, such as this one, there can be a shift in mathematics education research to gather and analyze the individual thinking of students with LD at rates similar to that of their peers without disability labels.

Qualitative research on students with LD may be minimal because some teachers and mathematics education researchers exhibit a deficit view of students with LD (Lambert, 2018). Rather than researching, identifying, and discussing the strengths of students with LD, students with LD are typically referred to by the ways in which they differ from students without disability labels. Researchers and educators tend to focus on what students with LD cannot do instead of highlighting what they can do. For instance, mathematics reform over the last thirty years has called for students to be active participants in their construction of mathematical knowledge through inquiry and discussion (Woodward & Montague, 2002). However, special educators typically question constructivism, the paradigm they equate with inquiry-based or discovery learning (Lambert & Sugita, 2016), because they believe that students with LD need explicit guidance to develop new mathematical knowledge (Jitendra, 2013). Furthermore, there is an underlying belief that pedagogical approaches that utilize discovery learning would lead “to even greater failure for students with learning disabilities” (Woodward & Montague, 2002, p. 92). Based on this belief, teachers are assuming, either consciously or unconsciously, that students with LD cannot use their prior knowledge to make connections and think about mathematical problems in an effort to build new mathematical knowledge. As a result, teachers may be limiting the mathematical potential of students with LD (Lambert, 2018). In her work, Lambert (2018) argued, “Considering the cognitive strengths of those with LD, it seems illogical to frame these learners as incapable of conceptual thinking” (p. 3). Rather, Lambert recommended that teachers should engage students with LD in standards-based instruction and provide them with the appropriate structure and support.

Educators may assume that it might be too cognitively challenging for students with LD to construct their own knowledge. As a result, teachers tend to rely on explicit instruction (Lambert, 2018). However, Lambert (2018) noted that students with LD develop new mathematical knowledge based on their previous understanding, and, as such, they deserve to have access to standards-based instruction. Not only did Lambert assert that students with LD should participate in inquiry-based instruction, but also the National Mathematics Advisory Panel (Geary et al., 2008) shared that explicit instruction should not be the only method used to teach students with LD in mathematics. The findings from this study suggest that students with LD do not drastically differ from that of students without disability labels in their understanding of linear functions. Thus, teachers should not restrict students' understanding by utilizing only explicit instruction. Instead, teachers should allow students with LD to reason, problem-solve, and complete complex mathematical tasks on algebraic concepts.

Findings from this study suggest that students with LD can articulate their learning needs, preferences, and mathematical content knowledge. In regard to instruction, participants explained that all students learn differently and that teachers should consider this when planning instruction and executing a lesson. Even though participants in this study preferred explicit instruction, they also liked it when their teachers showed them how to solve a problem in multiple ways. Additionally, participants found it advantageous when their teachers allowed them to solve tasks using whichever method they preferred. This notion of encouraging students to solve tasks using methods, skills, and meaningful strategies aligns with standards-based mathematics instruction. Researchers and educators can use this knowledge to begin to bridge the gap between explicit instruction and inquiry-based instruction. Instead of utilizing only explicit instruction, teachers can design inquiry-based instruction that incorporates instances of explicit instruction to support students' understanding at various points of the lesson. However, teachers should encourage students to make meaning of the larger task on their own or in a small group of their peers. In addition, teachers can connect procedures to problems that require a deeper conceptual understanding so that students build the logic and rationale as to why procedures are valid (Capraro & Joffrion, 2006). Mathematics co-teachers may wish to plan their inquiry-based instruction in accordance with students' knowledge and strengths while incorporating instances of explicit instruction to meet students' needs and preferences.

By listening to students when they explain their unique understanding, researchers and teachers can develop instructional strategies based on students' strengths and knowledge. For instance, results from this study suggest that teachers can utilize multiple representations, in particular graphical representations and a table of values, to help students with LD develop a conceptual understanding of linear functions. As such, teachers can make connections between representations to foster a deeper understanding of the rate of change, y-intercept, and algebraic equations in terms of the context of a real-world problem. With that said, not all real-world problems are meaningful to students. For instance, in Task 2, participants struggled when explaining the relationship between the number of greeting cards that Tanya made and the total amount she spent. Connecting their prior knowledge to make sense of the rate of change and y-intercept in Task 2 may have been problematic for participants because spending money to make greeting cards may not have been relatable to students.

Overall, students with LD have the knowledge, both real-world and mathematical, and can build on their knowledge to learn new mathematics concepts. To learn more about students with LD mathematical thinking, knowledge, and understanding of mathematical concepts, research could be conducted on other mathematical topics. Possible research topics could include ratios and proportions, geometric transformations, probability, and statistics. In addition, future research and instruction should focus on students with LD access to the standards-based curriculum, which simultaneously addresses their academic strengths, needs, and learning preferences. In particular, this research on instruction should incorporate classroom observations. While mathematical task interviews provide detailed data on individual student thinking, it is difficult to make sense of students' mathematical thinking and instructional preferences without information about the type of instruction students have received. Research that couples observations of teaching methods with student understanding can serve as a valuable source of information to help inform instruction.

For instruction to be effective for students with and without disability labels, mathematics ICT co-teachers need guidance and support. General education and special education teachers have beliefs about their pedagogy and instruction. In addition, educators have vast knowledge and experience on ways to plan, implement, and manage instruction. Mathematics education researchers, curriculum developers, and administration cannot simply tell teachers to implement an inquiry-based curriculum without appropriate professional development, particularly when supporting students with LD. Because little research on students with LD participating in a standards-based or inquiry-based curriculum exists (Lambert & Sugita, 2016), professional development on how to support students with LD may be limited. Curriculum developers may suggest common scaffolds for students with LD, such as utilizing graphic organizers, creating a word bank, or simplifying the task. For instance, in an inquiry-based curriculum utilized throughout the state in which this study was conducted, students were asked to find the value of *x* to make the equation $9(4 – 2x) – 3 = 4 – 6(3x – 5)$ linear. Within this curriculum, the only suggested scaffold provided for students with diverse learning needs on this task was to simplify the equation to $9(4 – 2x) – 3 = -18x$. Little information was given on how else to assist or instruct students with LD or other disability labels on this task. As such, there is a need for research on how to implement inquiry-based instruction within an ICT mathematics class effectively. The findings from this study suggest that educators can use a mix of instructional strategies within an inquiry-based lesson or curriculum to support students with LD, such as instances of explicit instruction. However, what does incorporating both inquiry-based and explicit instruction within the same lesson look like when implemented? How might teachers use their own knowledge and experience to gauge topics that students with LD can explore using their prior knowledge instead of other topics in which they might need explicit instruction? Rather than focusing on one instructional practice, how does using a mix of practices help to build the conceptual understanding of students with LD? Ultimately, teachers need resources and support to implement new instructional approaches in ICT mathematics classes. For that to be possible, more research is needed to determine effective practices, strategies, and examples.

Implications for Practice

As stated previously, students with LD are experts in their own classroom experience. Too often, classroom instruction in mathematics is structured so that teachers are the sole providers of feedback. Little time during a class period allows students to share their thoughts about the way in which teachers implement instruction. Not only do they have thoughts about their mathematics instruction, but also students with LD can clearly articulate these preferences with their teachers and mathematics education researchers. The only way that teachers will meet the needs of their students with LD is through giving students a voice. As such, it is imperative that mathematics education researchers and educators ask students with LD about their preferences for instruction directly, and they must continue to ask, as answers are not universal and will differ among contexts and students (Cook-Sather, 2002). Furthermore, after giving students the opportunity to share their thoughts, teachers must consider these needs when planning instruction. Educators can use insights from students with LD to modify and accommodate instruction that supports a conceptual development while still maintaining the academic rigor of the content, lesson, and task.

Rather than making assumptions based on disability labels or learning trajectories, teachers may want to learn the strengths of their students with LD. Mathematics educators and researchers should not automatically assume that inquiry-based instruction is too cognitively challenging for students with LD. Similarly, educators should not believe that students with LD cannot undertake more complex and abstract mathematical tasks or develop a conceptual understanding of algebraic concepts. Students with LD in this study recognized the essential concepts of a linear function and attempted to find the rate of change and y-intercept using procedural approaches. While solving for the rate of change and y-intercept, five of the six participants used mathematically valid approaches. However, some participants did make calculation errors when completing tasks. Although participants showed a limited conceptual understanding, educators can use students' strengths to help highlight mathematical connections. For instance, in this study, participants recognized that they could use coordinates from a graphical representation and a table of values to write a linear equation. Teachers can use this knowledge to demonstrate the relationship between various representations of linear functions, which can help students with LD develop a deeper understanding of the rate of change and y-intercept. As such, teachers may want to consider utilizing a mixture of instructional practices to educate students with LD. More importantly, teachers should try not to make assumptions about students with LD abilities to complete complex mathematical tasks or understand abstract algebraic concepts.

In this study, participants overwhelmingly shared that they preferred explicit instruction in mathematics. During their interview, participants highlighted that they liked when their teachers broke down new mathematical concepts into smaller steps. Additionally, participants found it advantageous for their learning when their co-teachers included guided practice and instances of repeated exposure during explicit instruction. Explicit instruction has been noted as a beneficial instructional practice for students with LD (Bryant et al., 2003; Jitendra, 2013; McLeskey et al., 2017), in which students, as a group, have shown statistically significant growth from pretest to posttest scores (Gersten et al., 2009). However, many studies that used explicit instruction as an intervention focused on mathematical skills or concepts at the elementary school level (Gersten et al., 2009). One argument against explicit instruction is that it may hinder an in-depth understanding of more abstract concepts (Lambert, 2018), such as linear functions. Because students are focused on following the steps of a procedure, it was unknown whether students knew when and how to apply a procedure to a task given at a later date or in a different form. In this study, five of the six participants attempted to apply a relevant procedure to find the rate of change and y-intercept. As some of the participants were entering 11th and 12th grade at the time of the study, the findings suggest that they recalled a procedure previously taught and applied it appropriately during the mathematical task interview. The use of procedures taught through explicit instruction can be a sound instructional practice for students with LD when teaching algebraic concepts. If students with LD prefer explicit instruction, teachers can incorporate instances of explicit instruction to meet their needs. With that said, when teaching procedures using explicit instruction, it is imperative that teachers make connections and use multiple representations to aid students in their development of a deeper understanding of the concept. While students can learn how to follow the steps of a procedure, they should also learn the rationale as to why the procedure works. Not only does understanding the relationship between the concept and procedure make mathematics more meaningful for students, but the hope is that this deeper understanding will foster students' awareness of when and how to apply this procedure on future tasks.

For co-teachers to succeed in meeting the needs of all students in an ICT class, co-teachers must have a solid understanding of the various co-teaching models. More importantly, both teachers must buy into the notion of co-teaching. Simply assigning a general education teacher and a special education teacher to an ICT class does not mean that classroom instruction will differ from a traditional mathematics class. Many ICT mathematics classes are still structured in a way in which the general education teacher is the main teacher, and the special educator provides one-on-one or small group assistance (King-Sears & Strogilos, 2018). Furthermore, in mathematics, inquiry-based instruction and explicit instruction are often viewed as opposing instructional practices. As such, co-teachers must overcome their own underlying beliefs about pedagogy and work together to design instruction that meets the needs of all students. However, timing and staffing constraints may make it difficult for co-teachers to work together to plan appropriate instruction. Rather than teachers relying on either inquiry-based instruction or explicit instruction, both co-teachers can share their valuable knowledge and resources with each other in an effort to incorporate various instructional approaches within lessons on a regular basis.

Although more and more students with LD are educated within a general education classroom, students with LD as a group are performing below their peers on standardized tests such as the National Assessment of Educational Progress (NAEP; Cortiella & Horowitz, 2014). As such, there is a need to explore and better understand students with LD experiences in mathematics classes and their content knowledge. Is there a potential disconnect between mathematics instruction and student performance? Are co-teachers of ICT mathematics classes planning instruction based on students' preferences and strengths? Furthermore, are co-teachers making conscious or unconscious assumptions about students with LD mathematics abilities and skills? Are external pressures such as standards-based accountability measures forcing teachers to ignore students' misunderstandings, rather than responsively adjusting and extending their instruction? As evident from this study, one promising way in which teachers and mathematics education researchers can support students with LD in mathematics is through interviewing students, listening to their preferences, and probing them to explain their mathematical knowledge of a concept. Participants in this study shared awareness of the notion that students think and learn differently. As such, participants wanted their teachers to take into consideration and honor these learning differences. Additionally, findings from this study suggest that not only can students with LD complete complex mathematical tasks that involve abstract algebraic concepts, but also their knowledge of linear functions did not drastically differ from the literature on students without disabilities. By giving voice and listening to students with LD, educators and researchers can make informed curriculum and instructional decisions. A deep conceptual understanding in mathematics can serve as a strong foundation for students with LD, which will allow them not only to gain entry but also to succeed in higher-level mathematics courses.

Limitations

In an attempt to reveal and highlight the voices of students with LD, interviews were used in this study to focus on the perspectives and knowledge of high school students with LD. Although using qualitative methods allowed a group of six high school students with LD to share their experiences with the researcher, this narrow scope was also a source of this study's limitations. Semi-structured interviews and mathematical task interviews maximize the opportunity for participants to voice their preferences and share their knowledge. However, interview studies are limited because data were self-reported by participants. Data were limited to the information that participants chose to disclose at the time of the interview. Additionally, as there were a limited number of participants in this study, results must be interpreted with caution. The results of both semi-structured interviews and mathematical task interviews cannot be generalized beyond the participants in this study. However, results can be applied and transferred to contexts that are similar. In addition to providing more validity to the findings of this study, future research that includes a larger pool of participants could show a greater depth of students with LD instructional preferences in mathematics and their content knowledge of linear equations. Furthermore, this study was not limited to only students with a mathematics disability such as dyscalculia. Instead, any student with a LD was eligible to participate in this study. The rationale for the inclusion of all students with LD is that in the urban school district where this study took place, students' Individualized Education Program (IEP) does not differentiate between the types of LD. Unless parents take their child for an evaluation outside of the school, the type of LD may remain unknown to educators, parents, and the student. Further research that focuses only on students with a mathematics LD may reveal other methods and instructional practices that teachers can use to support this particular group of students.

This study was designed for the semi-structured interview and the mathematical task interview to be conducted in person. At the time the study was conducted, the COVID-19 pandemic forced school closures and citywide shutdowns throughout the urban area in which this study was conducted. As a result, both the semi-structured interview and the mathematical task interview were conducted virtually through Zoom. The use of this platform may have influenced participants' responses. Additionally, interviews using Zoom were conducted in July and August, after participants in this study received approximately three months of remote instruction. Participants' experiences with remote instruction may have influenced their responses about their mathematics instruction and their knowledge about linear functions. In the original design of this study, the mathematical tasks were going to be printed by the researcher and given to each participant so that they could complete the tasks on paper using a pencil. Because mathematical task interviews were conducted virtually, the mathematical tasks were administered to participants using PearDeck, a Google Slides Add-On. The use of this platform allowed participants to view and complete the mathematical tasks on any electronic device, and it allowed the researcher to view their responses in real-time. However, the use of this platform may have limited participants' responses, as many of the participants were unfamiliar with the program. In addition, this platform may have been limiting because participants were accustomed to and comfortable with completing their mathematics work on paper. During the mathematical task interview, participants quickly learned the virtual platform, and they showed ease and proficiency when using it. Further research should be conducted with more participants using virtual platforms to explore the benefits of using these platforms as a method for educators and researchers to conduct mathematical task interviews.

References

Adu-Gyamfi, K., & Bosse, M. J. (2014). Processes and reasoning in representations of linear functions. *International Journal of Science and Mathematics Education*, *12*(1), 167-192. https://doi.org/10.1007/s10763-013-9416-x

Archer, A. L., & Hughes, C. A. (2011). *Explicit instruction: Effective and efficient teaching.* New York: The Guilford Press.

Ayalon, M., Watson, A., & Lerman, S. (2015). Functions represented as linear sequential data: Relationships between presentation and student responses. *Educational Studies in Mathematics, 90*, 321-339. https://doi.org/10.1007/s10649-015-9628-9

Beatty, R., & Bruce, C. D. (2012). Supporting students with learning disabilities to explore linear relationships using online learning objects. *PNA*, *7*(1), 21-39.

Bonner, S. M., & Chen, P. P. (2019). *Systematic classroom assessment: An approach for learning and self-regulation*. New York, NY: Routledge Press.

Brendle, J., Lock, R., & Piazza, K. (2017). A study of co-teaching identifying effective implementation strategies. *International Journal of Special Education, 32*(3), 538-550.

Brenner, M., Mayer, R., Moseley, B., Brar, T., Durán, R., Reed, B., & Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal*, *34*(4), 663-689.

Bryant, D. P., Hartman, P., & Kim, S. A. (2003). Using explicit and strategic instruction to teach division skills to students with learning disabilities. *Exceptionality*, *11*, 151–164. https://doi.org/10.4324/9781315046129-2

Capraro, M. M., & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology*, *27*(2-3), 147-164. https://doi.org/10.1080/02702710600642467

Clement, J. (2000). Analysis of clinical interviews: Foundations and model viability. In R. Lesh & A. Kelly (Eds.), *Handbook of research methodologies for science and mathematics education* (pp. 341-385). Hillsdale, NJ: Lawrence Erlbaum.

Commeyras, M. (1995). What can we learn from students' questions? *Theory into Practice, 43*(2), 101–106. https://doi.org/10.1080/00405849509543666

Cook, L., & Friend, M. (1995). Co-teaching: Guidelines for creating effective practices. *Focus on Exceptional Children*, *28*(3), 1–16. https://doi.org/10.17161/foec.v28i3.6852

Cook, S. C., & McDuffie-Landrum, K. (2020). Integrating effective practices into co-teaching:

Increasing outcomes for students with disabilities. *Intervention in School and Clinic, 55*(4), 221-229. https://doi.org/10.1177/1053451219855739

Cook, S. C., McDuffie-Landrum, K. A., Oshita, L., & Cook, B. G. (2017). Co-teaching for students with disabilities: A critical and updated analysis of the empirical literature. In J. M. Kauffman, D. P. Hallahan, & P. C. Pullen (Eds.), *The handbook of special education* (2nd ed., pp. 233–248). New York, NY: Routledge.

Cook-Sather, A. (2002). Authorizing students' perspectives: Towards trust, dialogue and change in education. *Educational Researcher*, *31*(4), 3–14. https://doi.org/10.3102/0013189x031004003

Common Core State Standards Initiative. (2010). Common Core State Standards for mathematics. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Cortiella, C., & Horowitz, S. H. (2014). *The state of learning disabilities: Facts, trends and emerging issues*. New York, NY: National Center for Learning Disabilities.

Costello, J., Toles, M., Spielberger, J., & Wynn, J. (2000). History, ideology and structure shape the organizations that shape youth. In N. Jaffe (Ed.), *Youth development: Issues, challenges, and directions* (pp. 185–231). Philadelphia: Public/Private Ventures.

Cramer, E. (2015). Shifting least restrictive environments in a large urban school district. *Journal of Urban Learning Teaching and Research, 11*(X), 40-49.

Creswell, J. (2015). *Educational research: Planning, conducting, and evaluating quantitative and qualitative Research* (5th ed.). Upper Saddle River, NJ: Pearson Education.

Dahl, K. (1995). Challenges in understanding the learner's perspective. *Theory into Practice, 43*(2), 124–130. https://doi.org/10.1080/00405849509543669

Davis, J. (2007). Real-world contexts, multiple representations, student-invented terminology, and y-intercept. *Mathematical Thinking and Learning, 9*(4), 387-418. https://doi.org/10.1080/10986060701533839

DeFur, S. H., & Korinek, L. (2010). Listening to student voices. *Clearing House, 83,* 15- 19.

DeSimone, J. R., & Parmar, R. S. (2006). Issues and challenges for middle school mathematics teachers in inclusion classrooms. *School Science and Mathematics, 106*, 338-348. https://doi.org/10.1111/j.1949-8594.2006.tb17754.x

Doabler, C. T., & Fien, H. (2013). Explicit mathematics instruction: What teachers can do for teaching students with mathematics difficulties. *Intervention in School and Clinic, 48,* 276–285. https://doi.org/10.1177/ 1053451212473151

Doabler, C. T., Fien, H., Nelson, N. J., & Baker, S. K. (2012). Evaluating three elementary mathematics programs for presence of eight research-based instructional design principles. *Learning Disability Quarterly, 35*, 200–211. https://doi.org/10.1177/0731948712438557

Dubinsky, E. (1993). Computers in teaching and learning discrete mathematics and abstract algebra. In D. L. Ferguson (Ed.), *Advanced educational technologies for mathematics and science* (pp. 525-583). New York: Springer-Verlag.

Ellis, A. B. (2007). Connections between generalizing and justifying: Students' reasoning with linear relationships. *Journal for Research in Mathematics Education, 38*(3), 194-229.

Ellis, A. B. (2009). Patterns, quantities, and linear functions. *Mathematics Teaching in the Middle School, 14*(8), 482-491.

Embury, D. C., & Kroeger, S. D. (2012). Let's ask the kids: Consumer constructions of co-teaching. *International Journal of Special Education, 27*(2), 102-112.

Filloy, E., & Sutherland, R. (1996). Designing curricula for teaching and learning algebra. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (Vol. 1, pp. 139-160). Dordrecht, The Netherlands: Kluwer

Fontana, K. C. (2005). The effects of co-teaching on the achievement of eighth grade students with learning disabilities. *The Journal of At-Risk Issues, 11*(2), 17–23.

Friend, M. (2008). Co-teaching: A simple solution that isn't simple after all. *Journal of Curriculum and Instruction, 2*, 9-19. https://doi.org/10.3776/joci.2008.v2n2p9-19

Friend, M. (2016). Welcome to co-teaching 2.0. *Educational Leadership, 73*(4), 16–22.

Friend, M., Cook, L., Hurley-Chamberlain, D., & Shamberger, C. (2010). Co-teaching: An illustration of the complexity of collaboration in special education. *Journal of Educational & Psychological Consultation, 20*(1), 9-27. https://doi.org/10.1080/10474410903535380

Gartner, A., & Lipsky, D. K. (1987). Beyond special education: Toward a quality system for all students. *Harvard Educational Review, 57*(4), 367-390. https://doi.org/10.17763/haer.57.4.kj517305m7761218

Geary, D. C., Boykin, A. W., Embretson, S., Reyna, V., Siegler, R., Berch, D. B., & Graban, J. (2008). Report of the national mathematics advisory panel. Washington, D.C.: U.S. Department of Education.

Gentilucci, J. L. (2004). Improving school learning: The student perspective. *Educational Forum, 68*(2), 133-141.

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79,* 1202–1242. https://doi.org/10.3102 /0034654309334431

Gillies, R. M. (2004). The effects of communication training on teachers' and students' verbal behaviors during cooperative learning. *International Journal of Educational Research, 41,* 257–279. https://doi.org/10.1016/j.ijer.2005.07.004

Goldin, G. A. (2000). A scientific perspective on task-based interviews in mathematics education research. In A. E. Kelly & R. A. Lesh (Eds.) *Handbook of research design in mathematics and science education* (pp. 517-546). Mahwah, NJ: Lawrence Erlbaum.

Graham, S., & Harris, K. R. (2009). Almost 30 years of writing research: Making sense of it all with *The Wrath of Khan. Learning Disabilities Research and Practice 24*(2), 58-68.

Hammersley, M., & Woods, P. (Eds.). (1984). *Life in school: The sociology of pupil culture.* Milton Keynes, UK: Open University Press.

Hart, K. M. (1981). *Children's understanding of mathematics.* London: John Murray.

Hattikudur, S., Prather, R. W., Asquith, P., Alibali, M. W., Knuth, E. J., & Nathan, M. (2012). Constructing graphical representations: Middle schoolers' intuitions and developing knowledge about slope and y-intercept. *School Science and Mathematics, 112*(4), 230-240. https://doi.org/10.1111/j.1949-8594.2012.00138.x

Hattie, J. (2009). *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement.* London, England: Routledge.

Herbert, S., & Pierce, R. (2012). Revealing educationally critical aspects of rate. *Educational Studies in Mathematical, 81*, 85-101. https://doi.org/10.1007/s10649-011-9368-4

Heritage, M. (2013). *Formative assessment in practice: A process of inquiry and action.* Cambridge, MA: Harvard Education Press.

Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics, 27*(1), 59-78. https://doi.org/10.1007/bf01284528

Heward, W. L., & Wood, C. L. (2013). *Improving educational outcomes in America: Can a low-tech, generic teaching practice make a difference?* Oakland, CA: Wing Institute for Evidence Based Practice. Retrieved from http://www.winginstitute.org/uploads/docs/2013WingSummitWH.pdf

Hitt, F. (1998). Difficulties in the articulation of different representations linked to the concept of function. *The Journal of Mathematical Behavior*, *17(*1), 123–134. https://doi.org/10.1016/s0732-3123(99)80064-9

Hoffer, W. W. (2016). *Developing literate mathematicians: A guide for integrating language and literacy instruction into secondary mathematics.* Reston, VA: National Council of Teachers of Mathematics, Inc.

Hord, C., & Newton, J. A. (2014). Investigating elementary mathematics curricula: Focus on students with learning disabilities. *School Science and Mathematics, 114,* 191-201.

Hughes, C. A., Morris, J. R., Therrien, W. J., & Benson, S. K. (2017). Explicit instruction: Historical and contemporary contexts. *Learning Disabilities Research & Practice, 32*(3), 140-148. https://doi.org/10.1111/ldrp.12142

Hunt, J. H., & Empson, S. B. (2015). Exploratory study of informal strategies for equal sharing problems of students with learning disabilities. *Learning Disability Quarterly, 38*(4), 208-220. https://doi.org/10.1177/0731948714551418

Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *Journal of Mathematical Behavior, 16*(2), 145-165. https://doi.org/10.1016/s0732-3123(97)90023-7

Hutchinson, N. L. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disability Quarterly, 16*(1), 34-63. https://doi.org/10.2307/1511158

Individuals with Disabilities Education Act (IDEA). (2004). PL 108-446, 20 U.S.C. § 1400 et seq. 2004.

Jackson, H. G., & Neel, R. S. (2006). Observing mathematics: Do students with EBD have access to standards-based mathematics instruction? *Education and Treatment of Children, 29*(4), 593-614.

Jitendra, A. K. (2013). Understanding and accessing standards-based mathematics for students with mathematics difficulties. *Learning Disability Quarterly, 36*(1), 4-8. https://doi.org/10.1177/0731948712455337

Jitendra, A. K., Griffin, C. C., McGoey, K., & Gardill, M. G. (1998). Effects of mathematical word problem solving by students at risk or with mild disabilities. *Journal of Educational Research, 91*, 345–355.

Johnston, P. H., & Nicholls, J. G. (1995). Voices we want to hear and voices we don't. *Theory Into Practice, 34*(2), 94-100. https://doi.org/10.1080/00405849509543665

Kaput, J. (1998). *Transforming algebra from an engine of inequity to an engine of mathematical power by "algebrafying" the K-12 curriculum*. Paper presented at the Algebra Symposium, Washington, DC.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal, 102*, 59–80. https://doi.org/10.1086/499693

Kendrick, M. (2010). Using student collaboration to foster progressive discourse. *The English Journal, 99*(5), 85-90.

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). New York: Macmillan.

King-Sears, M. E., & Strogilos, V. (2018). An exploratory study of self-efficacy, school belongingness, and co-teaching perspectives from middle school students and teachers in a mathematics co-taught classroom. *International Journal of Inclusive Education, 24*(2), 162-180. https://doi.org/10.1080/13603116.2018.1453553

Knuth, E. J. (2000). Student understanding of the Cartesian connection: An exploratory study. *Journal for Research in Mathematics Education, 31*(4), 500–507. https://doi.org/10.2307/749655

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equivalence and variable. *ZDM*, *37*(1), 68-76. https://doi.org/10.1007/978-3-642-17735-4_15

Kroesbergen, E. H., & Van Luit, J. E. H. (2003). Mathematics interventions for children with special needs: A meta-analysis. *Remedial and Special Education, 24*, 97-114. https://doi.org/10.1177/07419325030240020501

Lack, B., Swars, S. L., & Meyers, B. (2014). Low- and high-achieving sixth-grade students' access to participation during mathematics discourse. *The Elementary School Journal, 115*(1), 97-123. https://doi.org/10.1086/676947

Lambert, R. (2016). "When I am being rushed it slows down my brain": Constructing self-understandings as a mathematics learner. *International Journal of Inclusive Education, 21*(5), 521-531. https://doi.org/10.1080/13603116.2016.1251978

Lambert, R. (2018). "Indefensible, illogical and unsupported"; Countering deficit mythologies about the potential of students with learning disabilities in mathematics. *Education Sciences, 8*, 1-12. https://doi.org/10.3390/edusci8020072

Lambert, R., & Sugita, T. (2016). Increasing engagement of students with learning disabilities in mathematical problem-solving and discussion. *Support for Learning, 31*(4), 348-366. https://doi.org/10.1111/1467-9604.12142

Lambert, R., & Tan, P. (2016). Dis/ability and mathematics: Theorizing the research divide between special education and mathematics. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: University of Arizona.

Lambert, R., & Tan, P. (2017). Conceptualizations of students with and without disabilities as mathematical problem solvers in educational research: A critical review. *Education Sciences, 7*(2). https://doi.org/10.3390/educsci7020051

Leafstedt, J. M., Richards, C., LaMonte, M., & Cassidy, D. (2007). Perspectives on co-teaching: Views from high school students with learning disabilities. *Learning Disabilities: A Multidisciplinary Journal, 14*(3), 177-184.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1-64. https://doi.org/10.3102/00346543060001001

Legard, R., Keegan, J., & Ward, K. (2003). In-depth interviews. In J. Ritchie & J. Lewis (Eds.). *Qualitative research practice – a guide for social science students and researchers*, (pp. 138-169). Sage Publications, London.

Lewis, K. E., & Fisher, M. B. (2018). Clinical interviews: Assessing and designing mathematics instruction for students with disabilities. *Intervention in School and Clinic, 53*(5), 283-291. https://doi.org/10.1177/1053451217736864

Linsell, C. (2009). A hierarchy of strategies for solving linear equations. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 331-338). Palmerston North, NZ: MERGA.

Lobato, J., Ellis, A. B., & Muñoz, R. (2003). How "focusing phenomena" in the instructional environment afford students' generalizations. *Mathematical Thinking and Learning, 5*(1), 1–36. https://doi.org/10.1207/s15327833mtl0501_01

Logan, K., & Keefe, E. (1997). A comparison of instructional context, teacher behavior, and engaged behavior for students with severe disabilities in general education and self-contained elementary classrooms. *Journal of the Association for Persons with Severe Handicaps, 22*(1), 16-27. https://doi.org/10.1177/154079699702200102

Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice, 15*(1), 10–21. https://doi.org/10.1207/sldrp1501_2

Magiera, K., Smith, C., Zigmond, N., & Gebauer, K. (2005). Benefits of co-teaching in secondary mathematics classes. *TEACHING Exceptional Children, 37*(3), 20–24. https://doi.org/10.1177/004005990503700303

Manouchehri, A., & Enderson, M. C. (1999). Promoting mathematical discourse: Learning from classroom examples. *Mathematics Teaching in Middle School, 4*(4), 216-222.

Mastropieri, M. A., Scruggs, T. E., Bakken, J. P., & Whedon, C. (1996). Reading comprehension: A synthesis of research in learning disabilities. In T. E. Scruggs & M. A. Mastropieri (Eds.), *Advances in learning and behavioral disabilities* (Vol. 10B, pp. 201-227). Greenwich, CT: JAI Press.

Mastropieri, M. A., Scruggs, T. E., Graetz, J., Norland, J., Gardizi, W., & McDuffie, K. (2005). Case studies in co-teaching in the content areas: Successes, failures and challenges. *Intervention in School and Clinic, 40,* 260–270. https://doi.org/10.1177/10534512050400050201

Mathematics Assessment Project (MAP). 2015. Mathematics assessment resource service. http://map.mathshell.org/materials/index.php

McLeskey, J., Barringer, M. D., Billingsley, B., Brownell, M., Jackson, D., Kennedy, M., Lewis, T., Maheady, L., Scheeler, M. C., Winn, J., & Ziegler, D. (2017). *High-leverage practices in special education*. Arlington, VA: Council for Exceptional Children & CEEDAR Center.

Mendez, E. P., Sherin, M. G., & Louis, D. A. (2007). Multiple perspectives on the development of an eighth-grade mathematical discourse community. *The Elementary School Journal, 108*(1), 41-61. https://doi.org/10.1086/522385

Mitra, D. L. (2003). Student voice in school reform: Reframing student-teacher relationships. *McGill Journal of Education, 38*(2), 289-304.

Montague, M., Krawec, K., & Sweeny, C. (2008). Promoting self-talk to improve middle school students' mathematical problem solving. *Perspectives on Language and Literacy, 24*(2), 13-17.

Moschkovich, J., Schoenfeld, A., & Arcavi, A. H. (1993). Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. In T. A. Romberg, E. Fennema, & T. P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 69-100). Hillsdale, NJ: Erlbaum.

Murawski, W. W., & Lochner, W. W. (2011). Observing coteaching: What to ask for, look for, and listen for. *Intervention in School and Clinic, 46,* 174–183. https://doi.org/10.1177/1053 451210378165

National Center for Education Statistics. (2019). Public High School Graduation Rates. Retrieved from: https://nces.ed.gov/programs/coe/indicator_coi.asp

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all.* Reston, VA: National Council of Teachers of Mathematics.

Neill, K. (2021a). “Some kids understand things differently:” Perceptions of students with LD on their mathematics instruction in an integrated co-taught setting. Unpublished manuscript.

Neill, K. (2021b). Demonstrating their knowledge and understanding of linear functions: What do students with LD know?. Unpublished manuscript.

No Child Left Behind (NCLB) Act of 2001, Pub. L. No. 107-110, § 115, Stat. 1425 (2002).

Noble, T., Nemirovsky, R., Dimattia, C., & Wright, T. (2004). Learning to see: Making sense of the mathematics of change in middle school. *International Journal of Computers for Mathematical Learning, 9*(2), 109–167. https://doi.org/10.1023/B:IJCO.0000040891.50250.7e

Oehrtman, M., Carlson, M., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students’ function understanding. In M. Carlson & C. Rasmussen (Eds.), *Making the connection: research and teaching in undergraduate mathematics education* (pp. 27–42). Washington, DC: Mathematical Association of America. https://doi.org/10.5948/upo9780883859759.004

Oldfather, P. (1995). What’s needed to maintain and extend motivation for literacy in the middle grades. *Journal of Reading, 38*(6), 420–422.

Owen, R. L., & Fuchs, L. S. (2002). Mathematical problem-solving strategy instruction for third-grade students with learning disabilities. *Remedial and Special Education, 23,* 268–278. https://doi.org/10.1177/07419325020230050201

Pancsofar, N., & Petroff, J. G. (2016). Teachers’ experiences with co-teaching as a model for inclusive education. *International Journal of Inclusive Education, 20*(10), 1043–1053. https://doi.org/10.1080/13603116.2016.1145264

Patton, M. Q. (2002). *Qualitative research and evaluation methods.* Thousand Oaks, CA: SAGE Publications.

Patton, M. Q. (2015). *Qualitative research & evaluation methods: Integrating theory and practice: The definitive text of qualitative inquiry frameworks and options.* Thousand Oaks, CA: SAGE Publications.

Piaget, J. (1967). *Logique et connaissance scientifique.* Paris: Gallimard.

Piccolo, D. L., Harbaugh, A. P., Carter, T. A., Capraro, M. M., & Capararo, R. M. (2008). Quality of instruction: Examining discourse in middle school mathematics instruction. *Journal of Advanced Academics, 19*(3), 376-410.

Pierce, R., Stacey, K., & Bardini, C. (2010). Linear functions: Teaching strategies and students' conceptions associated with y = mx + b. *Pedagogies: An International Journal, 5*, 202-2015. https://doi.org/10.1080/1554480X.2010.486151

Powell, S. R., Fuchs, L. S., & Fuchs, D. (2013). Reaching the mountaintop: Addressing the common core standards in mathematics for students with mathematics difficulties. *Learning Disabilities Research & Practice*, *28*, 38-48. https://doi.org/10.1111/ldrp.12001

Rexroat-Frazier, N., & Chamberlin, S. (2019). Best practices in co-teaching mathematics with special needs students. *Journal of research in Special Education Needs, 19*(3), 173–183. https://doi.org/10.1111/1471-3802.12439

Riccomini, P. J., Morano, S., & Hughes, C. A. (2017). Big ideas in special education: Specially designed instruction, high-leverage practices, explicit instruction, and intensive instruction. *Council for Exceptional Children, 50*, 20-27. https://doi.org/10.1177/0040059917724412

Rice, D., & Zigmond, N. (2000). Co-teaching in secondary schools: Teacher reports of development in Australian and American classrooms. *Learning Disabilities Research and Practice*, *15*, 190–197. https://doi.org/10.1207/sldrp1504_3

Ross, P. A., & Braden, J. P. (1991). The effects of token reinforcement versus cognitive behavior modification on learning-disabled students' math skills. *Psychology in the Schools, 28*, 247–256. https://doi.org/10.1002/1520-6807(199107)28:3<247::aid-pits2310280310>3.0.co;2-e

Rudduck, J., & Flutter, J. (2000). Pupil participation and perspective: 'Carving a new order of experience'. *Cambridge Journal of Education, 30*(1), 75–89. https://doi.org/10.1080/03057640050005780

Saldaña, J. (2009). *The coding manual for qualitative researchers*. London: Sage.

Scheurmann, A. M., Deschler, D. D., & Schumaker, J. B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly, 32*, 103–120. https://doi.org/10.2307/27740360

Scruggs, T. E., Mastropieri, M. A., & McDuffie, K. A. (2007). Co-teaching in inclusive classrooms: A metasynthesis of qualitative research. *Exceptional Children, 73, 392-416.* https://doi.org/10.1177/001440290707300401

Shepherd, M. (2012). A quantitative discourse analysis of student-initiated checks of understanding during teacher-fronted lessons. *Linguistics and Education, 23*(1), 145-159. https://doi.org/10.1016/j.linged.2011.09.003

Sileo, J. M., & van Garderen, D. (2010). Creating optimal opportunities to learn mathematics: Blending co-teaching structures with research-based practices*. TEACHING Exceptional Children. 42*(3), 14-21. https://doi.org/10.1177/004005991004200302

Steele, D. F. (2001). Using sociocultural theory to teach mathematics: A Vygotskian perspective. *School Science and Mathematics, 101*(8), 404-416. https://doi.org/10.1111/j.1949-8594.2001.tb17876.x

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2015).Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning, 10,* 313-340. https://doi.org/10.3102/978-0-935302-43-1_29

Strickland, T. K., & Maccini, P. (2013). The effects of the concrete-representational-abstract integration strategy on the ability of students with learning disabilities to multiply linear expressions within area problems. *Remedial and Special Education*, *34*(3), 142–153. https://doi.org/10.1177/0741932512441712

Strogilos, V., & King-Sears, M. E. (2019). Co-teaching is extra help and fun: Perspectives on co-teaching from middle school students and co-teachers. *Journal of Research in Special Educational Needs, 19*(2), 92-102. https://doi.org/10.1111/1471-3802.12427

Stump, S. L. (2001). Developing preservice teachers' pedagogical content knowledge of slope. *The Journal of Mathematical Behavior, 20*(2), 207-227. https://doi.org/10.1016/S0732-3123(01)00071-2

Swanson, H. L. (2001). Searching for the best model of instructing students with learning disabilities. *Focus on Exceptional Children, 34*(2), 1-14. https://doi.org/10.17161/fec.v34i2.6785

Teuscher, D., & Reys, R. E. (2010). Slope, rate of change, and steepness: Do students understand these concepts? *Mathematics Teacher, 103*(7), 519-524.

Thompson, P. W., & Carlson, M. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). Reston, VA: National Council of Teachers of Mathematics.

Ulusoy, F., & Argun, Z. (2019). Secondary school students' representations for solving geometric word problems in different clinical interviews. *International Journal of Education in Mathematics, Science, and Technology*, *7*(1), 73-91. https://doi.org/10.18404/ijemst.328341

United States Department of Education. (2010). *A blueprint for reform: The reauthorization of the Elementary and Secondary Education Act*. Washington, DC: U.S. Department of Education, Office of Planning, Evaluation and Policy Development.

United States Department of Education. (2018). Sec. 300.8 (c) (10) Individuals with Disabilities Education Act. Sec. 300.8 (c) (10). https://sites.ed.gov/idea/regs/b/a/300.8/c/10

Urquhart, C. (2013). *Grounded theory for qualitative research: A practical guide.* London; Thousand Oaks, CA: Sage.

Vaughn, S., Gersten, R., & Chard, D. J. (2000). The underlying message in LD intervention research: Findings from research syntheses. *Exceptional Children 67*(1), 99-114. https://doi.org/10.1177/001440290006700107

Villa, R. A., Thousand, J. S., & Nevin, A. I. (2008*). A guide to co-teaching: Practical tips for facilitating student learning* (2nd ed.). Thousand Oaks, CA: Corwin Press.

von Glaserfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. London: Falmer.

Vygotsky, L. S. (1978). *Mind in Society* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Cambridge, MA: Harvard University Press.

Walther-Thomas, C. S. (1997). Co-teaching experiences: The benefits and problems that teachers and principals report over time. *Journal of Learning Disabilities*, *30*, 395–408. https://doi.org/10.1177/002221949703000406

Wang, Y., Barmby, P., & Bolden, D. (2017). Understanding linear function: A comparison of selected textbooks from England and Shanghai. *International Journal of Science and Mathematics Education*, *15*, 131-153. https://doi.org/10.1007/s10763-015-9674-x

Watt, S., Watkins, J., & Abbitt, J. (2016). Teaching algebra to students with learning disabilities: Where have we come and where should we go? *Journal of Learning Disabilities*, *49*(4), 1-11. https://doi.org/10.1177/0022219414564220

Wehmeyer, M. L. (2006). Beyond access: Ensuring progress in the general education curriculum for students with severe disabilities. *Research & Practice for Persons with Severe Disabilities*, *31*, 322–326. https://doi.org/10.1177/154079690603100405

Weibe Berry, R. A. W., & Namsook, K. (2008). Exploring teacher talk during mathematics instruction in an inclusion classroom. *The Journal of Education Research, 101*(6), 363-377. https://doi.org/10.3200/JOER.101.6.363-378

Weiss, M. P., & Lloyd, J. W. (2002). Congruence between roles and actions of secondary special educators in co-taught and special education settings. *The Journal of Special Education, 36*(2), 58-68. https://doi.org/10.1177/00224669020360020101

Wilkie, K. J., & Ayalon, M. (2018). Investigating years 7 to 12 students' knowledge of linear relationships through different contexts and representations. *Mathematics Education Research Journal*, *30*, 499-523. https://doi.org/10.1007/s13394-018-0236-8

Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities—A Contemporary Journal*, *3*(2), 49–60.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, *18,* 121–131. https://doi.org/10.1111/1540- 5826.00068

Woodward, J., Monroe, K., & Baxter, J. (2001). Enhancing student achievement on performance assessments in mathematics. *Learning Disability Quarterly*, *24*, 33-46. https://doi.org/10.2307/1511294

Woodward, J., & Montague, M. (2002). Meeting the challenge of mathematics reform for students with LD. *The Journal of Special Education, 30*(2), 89-101. https://doi.org/10.1177/00224669020360020401

Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word problem-solving instruction on middle school students with learning problems. *Journal of Special Education*, *39*, 181–192.

Zahner, W. (2015). The rise and run of a computational understanding of slope in a conceptually focused bilingual algebra class. *Educational Studies Mathematics*, *88*, 19-41. https://doi.org/10.1007/s10649-014-9575-x

Zigmond, N., & Matta, D. (2004). Value added of the special education teacher on secondary school co-taught classes. In T. E. Scruggs & M. A. Mastropieri (Eds.), *Advances in learning and behavioral disabilities: Vol 17. Research in secondary schools* (pp. 55-76). Oxford, UK: Elsevier. https://doi.org/10.1016/s0735-004x(04)17003-1

APPENDIX A

IRB APPROVAL FORM

University Integrated Institutional Review Board
205 East 42nd Street
New York, NY 10017
http://www.cuny.edu/research/compliance.html

Approval Notice
Initial Application

07/21/2020

Kayla Neill,
Hunter College

RE: IRB File #2019-1106
Understanding the Perceptions and Thought Processes of Students with Disabilities in Mathematics

Dear Kayla Neill,

Your Initial Application was reviewed and approved on 07/21/2020. You may begin this research.

Please note the following information about your approved research protocol:

Protocol Approval Period:	07/21/2020
Protocol Risk Determination:	Minimal
Expedited Categor(ies):	(6) Collection of data from voice, video, digital, or image recordings made for research purposes.; (7) Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies. (NOTE: Some research in this category may be exempt from the HHS regulations for the protection of human subjects. 45 CFR 46.101(b)(2) and (b)(3). This listing refers only to research that is not exempt.);

Documents / Materials:

Type	Description	Version #	Date
Curriculum Vitae	CITI Requirements - Kayla Neill	1	12/04/2019
Curriculum Vitae	CITI Requirements - Dr. Nicora Placa	1	12/11/2019
Telephone Screening Script	UPTP_PhoneScreeningScript1.docx	1	02/04/2020
Advertisement	UPTP_RecruitmentFlyer.docx	1	02/07/2020
Survey(s)	UPTP_MemberCheckQualtrics.docx	1	02/18/2020

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Child Assent Document	UPTP_AdolescentAssent.docx	1	02/19/2020
Informed Consent Document	UPTP_ParentPermissionAdolescent.docx	1	02/19/2020
Interview Question(s)	UPTP_MathematicalTaskRemote.pdf	1	06/03/2020

Please remember to:

- Use **the IRB file number** 2019-1106 on all documents or correspondence with the IRB concerning your research protocol.

- Review and comply with CUNY Human Research Protection Program policies and procedures.

- The IRB has the authority to ask additional questions, request further information, require additional revisions, and monitor the conduct of your research and the consent process.

- Any modifications to currently approved research must be submitted to and approved by the CUNY-UI IRB before implementation.

If you have any questions, please contact:
Alicia Caldwell
ac2768@hunter.cuny.edu

APPENDIX B

RECRUITMENT FLYER

HUNTER
The City University of New York
695 Park Avenue, New York, N.Y. 10065
http://www.hunter.cuny.edu

Mathematics Research Opportunity

You are invited to join a research study:

Linear equations is an important topic to a student's success in algebra and other mathematics classes. This study will explore your knowledge of linear equations and your thoughts about your mathematics education. This study hopes to improve mathematics instruction for students with disabilities.

What Will We Do?
You will complete a few math problems with the researcher. Also, the researcher will ask you to answer a few questions about your mathematics teachers and their instruction during class. In all, this will take about 1 to 2 hours of your time.

Who Can Participate? You can participate if …

- You are in 9th, 10th, 11th or 12th grade
- You are between the ages of 13 and 17
- You are in an Integrated Co-Taught Class (2 teachers)

How Can I Participate?
Ask your parent and/or guardian to:

Contact [redacted]

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Protocol: 2019-1106
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Expires:

APPENDIX C

INTERNET SCREENING SURVEY

THE CITY UNIVERSITY OF NEW YORK
Hunter College
Department of Curriculum and Teaching

ELIGIBILITY SCREENING SCRIPT - PHONE

Title of Research Study:	Understanding the Perceptions and Thought Processes of Students with Disabilities in Mathematics
Principal Investigator:	Kayla Neill CUNY Hunter Doctorate Candidate, Instructional Leadership
Faculty Advisor:	Nicora Placa, Ph.D, Mathematics Education Assistant Professor

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Thank you for talking to me about my research. This research study will explore what students think about mathematics problems on linear functions and their perceptions about their mathematics education. I would like to ask you a few questions to determine whether your child is eligible to participate in this research. Would you like to continue with the screening?

Instruction: If yes, continue with the screening. If no, thank the person and end conversation.

The screening will take about 10 minutes. The survey will ask you some questions about your child's grade, his/her Individualized Education Program, and his/her disability. You do not have to answer any questions you do not wish to answer or are uncomfortable answering, and you may stop at any time. Your participation in the screening is voluntary.

I will make my best efforts to keep your answers and your child's information confidential. No one except for the research team will have access to these answers. If your chid qualifies for the research and he/she decides to participate, this information will be kept with the research record. It will be stored using codes to protect your child's privacy. If your child does not qualify for the study, the information that you provided during the screening will be destroyed.

Would you like to continue with the screening?

Instruction: If yes, continue with the screening. If no, thank the person and hang-up.

1. What grade is your child in?

    - If the child is in 9th, 10th, 11th, or 12th grade, the child is eligible for the study

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University Integrated IRB
Protocol: 2019-1106
Approved: 07/21/2020
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- If not, the child is not eligible as this study only include students from 9th, 10th, 11th, or 12th grade

Instruction: If not eligible, read: Thank you for your interest and your child's interest, but your child is not eligible for this study.

2. How old is your child?

   - If the child is 13 – 17, the child is eligible for the study

   - If not, the child is not eligible as this study only include students from the ages of 13 to 17 that are in 9th through 12th grade.

Instruction: If not eligible, read: Thank you for your interest and your child's interest, but your child is not eligible for this study.

3. Does your child have an Individualized Education Program (IEP)?

   - If the answer is **yes** and the parent of the child identifies the child as having an IEP, the child is eligible for the study

   - If the answer is **no**, the child is not eligible as this study only include students with IEPs

Instruction: If not eligible, read: Thank you for your interest and your child's interest, but your child is not eligible for this study.

4. Does your child have a Learning Disability?

   - If the parent of the child identifies the child as having a Learning Disability, the child is eligible for the study

   - If not, the child is not eligible as this study only include students with IEPs

Instruction: If not eligible, read: Thank you for your interest and your child's interest, but your child is not eligible for this study.

5. Is your child enrolled in an Integrated Co-Taught (ICT) class for mathematics? (2 years, 1 general education and 1 special education)

   - If the parent says yes that the child is enrolled in an ICT class for mathematics, the child is eligible for the study

   - If not, the student is not eligible as this study only include students with IEPs in ICT classes

Instruction: If not eligible, read: Thank you for your interest and your child's interest, but your child is not eligible for this study.

Thank you for answering the screening questions.

Instruction: Indicate whether the child is eligible; requires additional screening; or is not eligible and explain why. Use the following t-chart to help determine eligibility, if needed:

Eligible	Not Eligible
1. In 9th, 10th, 11th, or 12th grade **and** between the ages of 13 and 17 2. Identifies as having an Individualized Education Program 3. Identifies as having a Learning Disability 4. Is enrolled in an Integrated Co-Taught class (2 teachers)	1. In kindergarten through 8th grade or have graduated from high school 2. 12 years or young or 18 years or older 3. Does not identify as having an IEP 4. Does not identify has a Learning Disability 5. Is not enrolled in an Integrated Co-Taught Class

Instruction: If **eligible** at this point, read: Thank you for your interest and your child's interest in the survey. Your child is eligible to participate in this study.

Instruction: If not eligible at this point, read: Thank you for your interest and your child's interest in the survey, but your child is not eligible to participate in this study. End the conversation and hang-up.

Do you have any questions about the screening or the research? I am going to give you a telephone number to call and an email address if you have any questions later. Do you have a pen? If you have questions about the research screening, you may call Kayla Neill at [redacted]

If you have questions about your child's rights as a research participant, or if you wish to voice any problems or concerns to someone other than the researchers, please call CUNY Research Compliance Administrator at 646-664-8918.

Thank you again for your willingness to answer our questions.

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Protocol: 2019-1106
Approved: 07/21/2020
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APPENDIX D

PARENT PERMISSION AND ADOLESCENT ASSENT FORMS

THE CITY UNIVERSITY OF NEW YORK
Hunter College
Department of Curriculum and Teaching

PARENTAL PERMISSION FOR ADOLESCENT (AGE 13-17) TO PARTICIPATE IN A RESEARCH STUDY

Title of Research Study:	Understanding the Perceptions and Thought Processes of Students with Disabilities in Mathematics
Principal Investigator:	Kayla Neill CUNY Hunter Doctorate Candidate, Instructional Leadership
Faculty Advisor:	Nicora Placa, Ph.D, Mathematics Education Assistant Professor

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Your child is being asked to join a research project because he/she is a 9th, 10th, 11th, or 12th grader, between the ages of 13 and 17, and you have stated that your child has an Individualized Program (IEP) and a Learning Disability (LD).

Purpose:
This research is being done to understand what students think about mathematics problems. If your child participates in this project, he/she will be asked to solve a few math problems and share his/her thinking aloud. Another purpose of this project is to understand what students think about how they learn mathematics best. Your child may want to participate in this project to share his/her knowledge and thoughts about his/her education. However, your child might not want to participate in this project if he/she does not feel comfortable sharing his/her thinking while completing mathematics problems.

Key Information:

- Consent is needed for this research and participation is voluntary.
- The purpose of this research is to understand what students with Learning Disabilities know about linear functions and their thoughts about their instruction in mathematics. Participation in this project will be no more than 2 hours. Participants will complete a short survey and participate in a two-part interview. Participants can choose to participate in a member check.
- During the interview, the participant may feel uncomfortable. Because the two-part interview will be audiotaped, there is a chance that your child's identity may be revealed by their voice in these recordings.

- The interviews will take place at the public library. While the researcher intends to reserve a private room to conduct the interview, a private room may not be available. The interview will then take place somewhere in the library. Because the interviews will take place within a public library, there is a chance for a breach in confidentiality. The researcher will try to minimize a chance of a breach in confidentiality by finding a secluded area and re-arranging furniture. However, the researcher cannot ensure that complete confidentiality is maintained.

- There are no direct benefits for the participant.

- The participant can choose not to answer any question(s) during the interview if it causes discomfort.

Procedures:
If you agree to allow your child to participate in this research study, we will ask your child to do the following:

- Participate in a two-part interview. During the first part of the interview, your child will complete a few mathematical problems while explaining his/her thinking to the interviewer. It should take between 15 to 30 minutes. The second part of the interview will take place right after the first part. This part of the interview will take between 20 and 30 minutes. The second part of the interview will ask questions about the types of teaching and activities your child finds helpful for your learning. Both parts of the interview will be audio recorded.

- Participate in a member check. Even though your child will be asked to review the researcher's findings and provide input on those findings, this procedure is voluntary. If your child chooses to complete this procedure, you will be sent a link to an online survey with the findings. The researcher will ask you to share this link with your child and have your child complete a brief questionnaire online. This will include a series of yes or no questions about the findings. This procedure is not required to participate in the study. If your child chooses to review the researcher's findings, it will take no more than 15 minutes.

Audio Recording
Both parts of the two-part interview will be audio recorded, so that the researcher can type up and review your answers at a later date. Your child cannot participate in this study if he/she do not consent to audio recording.

Your child has the right to delete any part of the audio recording. To request to listen to the audiotape, your child can ask the researcher to listen to the audio tape at any point during the interview or at the end. To request to delete part of the interview, your child can tell the researcher which part he/she wishes to delete. If your child feels uncomfortable with this, your child can tell you that he/she wishes to listen to the audio recording and delete part of it. Then, you can contact me via email or phone and request that part of the interview be deleted.

Time Commitment:
Your child's participation in this research study is expected to last for a total of one to two hours.

Potential Risks or Discomforts:

- One possible risk is that your child may feel uncomfortable when being interviewed. The researcher intends to make this a positive experience for your child.

- Your child can decide not to answer any question(s) during the interview.

- Please know this study is not to evaluate your child, but to understand his/her thoughts about linear functions as well as his/her feelings towards your education in mathematics.

- While the data in this study will be kept confidential and your identity will never be revealed, a breach of confidentiality can occur when audio recording is involved. In addition, a breach of confidentiality may occur during the interview, as it is taking place at a public library. Please know that the researcher will take every step to protect your child's identity in this study as outlined in the Confidentiality section below.

Potential Benefits:

- Your child will not directly benefit from your participation in this research study.

- By participating in this study, your child is helping to better the education of students with Individualized Education Programs. The participant is giving the researcher the opportunity to understand his/her point of view and his/her understanding of linear functions. The researcher hopes to use this information to identify potential learning opportunities and improve mathematics instruction for students with Individualized Education Programs.

Payment for Participation:

Your child will not receive any payment for participating in this research study.

Confidentiality:

We will make our best efforts to maintain confidentiality of any information that is collected during this research study, and that can identify your child. We will disclose this information only with your permission or as required by law.

The researcher will make her best efforts to maintain confidentiality of any information that is collected during this research study, and that can identify your child. I will disclose this information only with your permission or as required by law.

The researcher will protect your child's confidentiality by keeping all data in locked files. This parent permission form, the adolescent assent form, and any other documents in which you or your child's name appears will be kept in a locked file separate from data. The audio files will be secured in a locked filing cabinet at CUNY Hunter. After the audio files have been transcribed by the researcher, the files will be destroyed. Only the research team will have access to these files. Your child has the right to review the audio recording taken as part of this research to determine whether they should be edited or erased in part. Once the research is complete, the data will be retained in the same locked filing cabinets for three years, and then it will be destroyed. It will not be saved for future use.

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The results of this project may be published in a journal and/or shared at meetings without naming your child as a participant. Your child's name will not be used when the results of the project are reported. Specific quotes from the interviews may be reported. However, a false name will be used. No identifying information will be shared. Your child's records will be kept completely confidential according to current legal requirements. They will not be revealed unless required by law, or as described in this form. The researcher, authorized CUNY staff, and government agencies that oversee this type of research may have access to research data and records in order to monitor the research. Research records provided to authorized, non-CUNY individuals will not contain identifiable information about you. Publications and/or presentations that result from this project will not identify your child by name.

The researcher intends to reserve a private room at a library to conduct the interview. However, due to scheduling, this may not always be possible. If a private room is not available, the interview will take place in a public space in the library. If this is the case, the researcher will find a secluded area that is semi-private. The researcher will re-arrange the furniture, so that the face and voice of your child cannot be easily identified by others in the library. However, because this is still a public space, complete confidentiality cannot be maintained.

Under New York State law, the researcher is a mandated reporter. This means that the researcher will not tell anyone what your child tells the researcher unless there is something that could be dangerous to your child or someone else. If your child tells the researcher that someone is or has been hurting your child, the researcher may have to tell that to people who are responsible for protecting children so they can make sure your child is safe.

The research team, authorized CUNY staff, and government agencies that oversee this type of research may have access to research data and records in order to monitor the research. Research records provided to authorized, non-CUNY individuals will not contain identifiable information about your child. Publications and/or presentations that result from this study will not identify your child by name.

Participants' Rights:

- Your child's participation in this research study is entirely voluntary. If you decide not to allow your child to participate, there will be no penalty to your child, and your child will not lose any benefits to which he/she is otherwise entitled.

- Your child's decision to not participate in this project will in no way impact his/her academic standing at your school or the services that your child is receiving. Participation in this study will also not impact your child's privileges at the agency in which your child attends after school or on weekends (e.g. The Boys and Girls Club).

- Your child can decide to withdraw his/her consent and stop participating in the research at any time, without any penalty. Any data that may have been collected will be destroyed.

Questions, Comments or Concerns:

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If you have any questions, comments or concerns about the research, you can talk to one of the following research team members:

Principal Investigator Kayla Neill [redacted]

Faculty Advisor Nicora Placa [redacted]

If you have questions about your child's rights as a research participant, or you have comments or concerns that you would like to discuss with someone other than the research team, please call the CUNY Research Compliance Administrator at 646-664-8918 or email hrpp@cuny.edu. Alternatively, you may write to:

CUNY Office of the Vice Chancellor for Research
Attn: Research Compliance Administrator
205 East 42nd Street
New York, NY 10017

Permission for Audio Recording

If you agree to audio recording, please indicate this below.

_______ I agree to audio recording

_______ I do **NOT** agree to audio recording

Permission for the Use of Quotes in Publications/Presentations

If you agree to the use of quotes in publications/presentations, please indicate this below.

_______ I agree to the use of quotes in publications/presentations

_______ I do **NOT** agree to the use of quotes in publications/presentations

Permission to Contact for a Member Check

If you agree to allow the researcher to contact you to have your child participate in a member check, please indicate this below.

_______ I agree to allow the researcher to contact me for a member check

_______ I do **NOT** agree to allow the researcher to contact me for a member check

Signature of Parent(s) or Legal Guardian:

If you give permission for your child to participate in this research study, please sign and date below. You will be given a copy of this form to keep.

______________________________ ______________________________
Printed Name of Parent or Legal Guardian | Printed Name of Child Participant

______________________________ ______________
Signature of Parent or Legal Guardian | Date

Signature of Individual Obtaining Parental Permission

____
Printed Name of Individual Obtaining Parental Permission

______________________________ ______________
Signature of Individual Obtaining Parental Permission | Date

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THE CITY UNIVERSITY OF NEW YORK
Hunter College
Department of Curriculum and Teaching

ADOLESCENT ASSENT (AGES 13-17) TO PARTICIPATE IN A RESEARCH STUDY

Title of Research Study:	Understanding the Perceptions and Thought Processes of Students with Disabilities in Mathematics
Principal Investigator:	Kayla Neill CUNY Hunter Doctorate Student, Instructional Leadership
Faculty Advisor:	Nicora Placa, Ph.D., Mathematics Education Assistant Professor

---

1. My name is Kayla Neill.

2. I am asking you to take part in a research study because I am trying to learn more about what you know about linear functions and what you think about your mathematics instruction in an Integrated Co-Taught (ICT) class.

3. If you agree to be in this study, you will be asked to complete a few mathematics problems. When you are solving each math problem, I will encourage you to explain your thinking out loud, and I may ask you some questions about how you solved each problem. After you have finished the mathematical problems, you will also be asked to answer some questions about your mathematics instruction in your ICT class. You will be asked questions that will invite you to share your thoughts about your mathematics education with me.

4. You may feel uncomfortable when being interviewed. I plan to make this an enjoyable experience and one that gives you the chance to share your thoughts. You can choose not to answer any question(s) during the interview. The interviews will take place at a public library. People may be able to see and identify you. Your complete confidentiality cannot be maintained. However, I plan to create a space in which your answers cannot be heard and your face cannot be easily seen by others in the library.

5. The interview will be audio recorded. I will tell you when I am starting the audio recording before I ask the first interview question. I will tell you when I am stopping the audio recording after you have finished answering the last interview question. You can ask me to delete any part of the interview. You can ask me to delete part of the audio recording immediately after you shared information that you want to delete or you can ask at the end of the interview. You can also ask your parent to contact the researcher if you want to delete part of the interview. The audio recording will be deleted after I have typed up your responses.

6. You will not receive anything for participating in this study. However, by sharing your knowledge and your thoughts, you are helping to better the education of other students with Learning

Disabilities. You may not want to participate in this study if you do not feel comfortable sharing your thoughts or completing a few mathematics problems.

7. You can talk this over with your parents before you decide whether or not to participate. We will also ask your parents to give their permission for you to take part in this study. But even if your parents say "yes" you can still decide not to do this.

8. If you don't want to be in this study, you don't have to participate. Remember, being in this study is up to you and no one will be upset if you don't want to participate or even if you change your mind later and want to stop.

9. You can ask any questions that you have about the study. If you think of a question later, you can call me at [redacted] or ask me next time.

10. Signing your name at the bottom means that you agree to be in this study. You and your parents will be given a copy of this form after you have signed it.

If you want to participate in this research, you can write your name on the line below:

____

If you agree that it is all right with you to be audio recorded during the interview, print your name on the line below:

____

APPENDIX E

SEMI-STRUCTURED INTERVIEW QUESTIONS

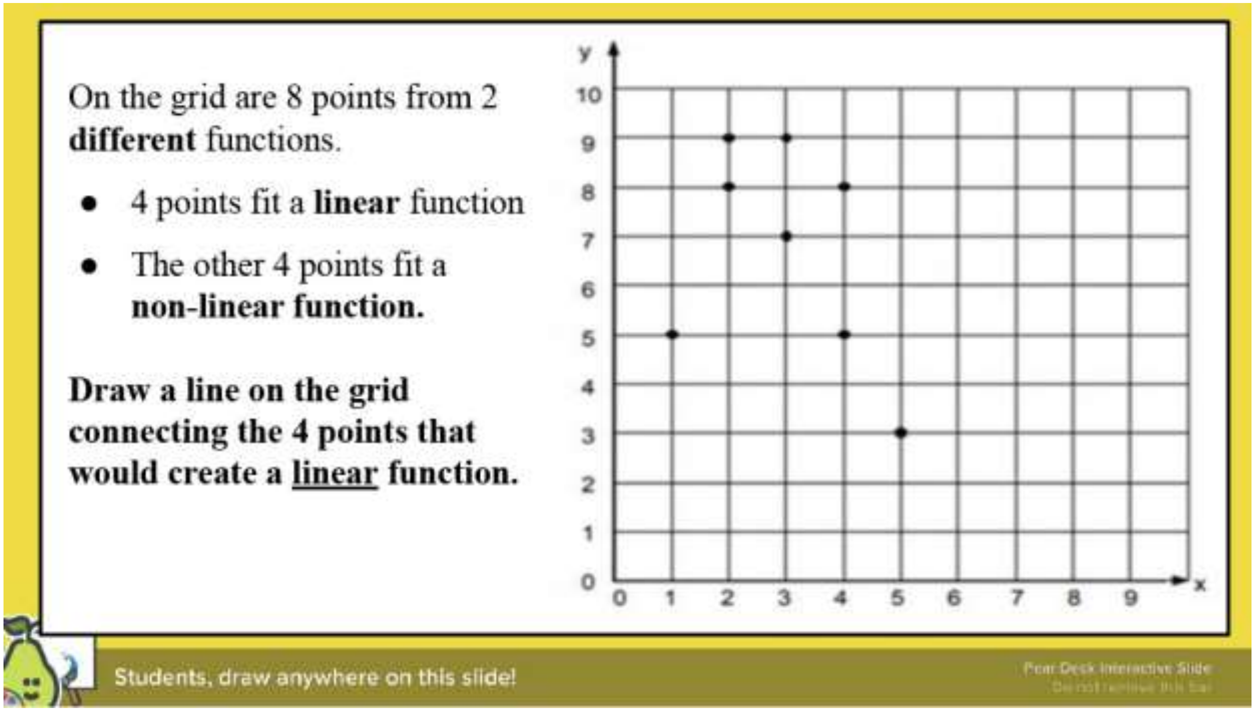
Interview Questions with Probing Questions

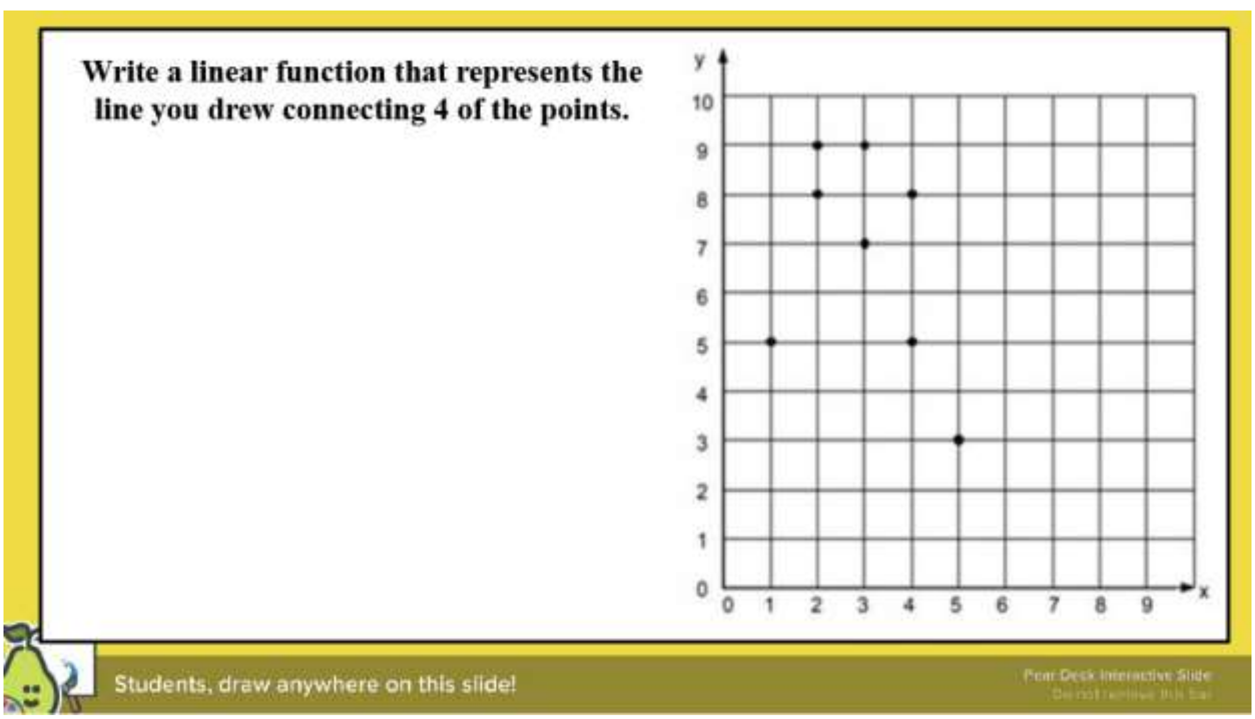
Interview Question	Probing Questions
How do you feel about your current math class?	What are some things that you like about your teachers? What are some things that you dislike about your teachers? Is the class easy? Hard? Why? Do you feel like you can be successful in this class? Why or why not?
How do your teachers structure their mathematics class?	Describe how class starts. Explain the way that your teachers introduce new material What types of work do you complete during class? Group activities? Worksheets? Projects? How do your teachers end class?
Describe some of the things that your mathematics teachers do that you find most helpful when learning math concepts.	Why do you think it is helpful? Describe the type of instruction and problems/activities you find helpful. What are some things that your teachers do that you do not find helpful? Name one or two things that you wish your teachers would do to support you in math class.
Is there anything that you would change about your current teachers' instruction in mathematics?	Describe how you would change it. Why would you change it?
Thinking back to all of your math classes and teachers in the past, which do you think were the best at teaching you math concepts and why?	Describe the type of teaching in math class that you like best. Name the types of activities and instruction you believe you need in order to be successful in math class. Do you find it helpful for your teacher to show you how to complete the task first or for you to be given a task and try to figure it out on your own? Why? Do you think it is helpful for your teachers to give you reference points to support your understanding and learning? Why or why not?

APPENDIX F

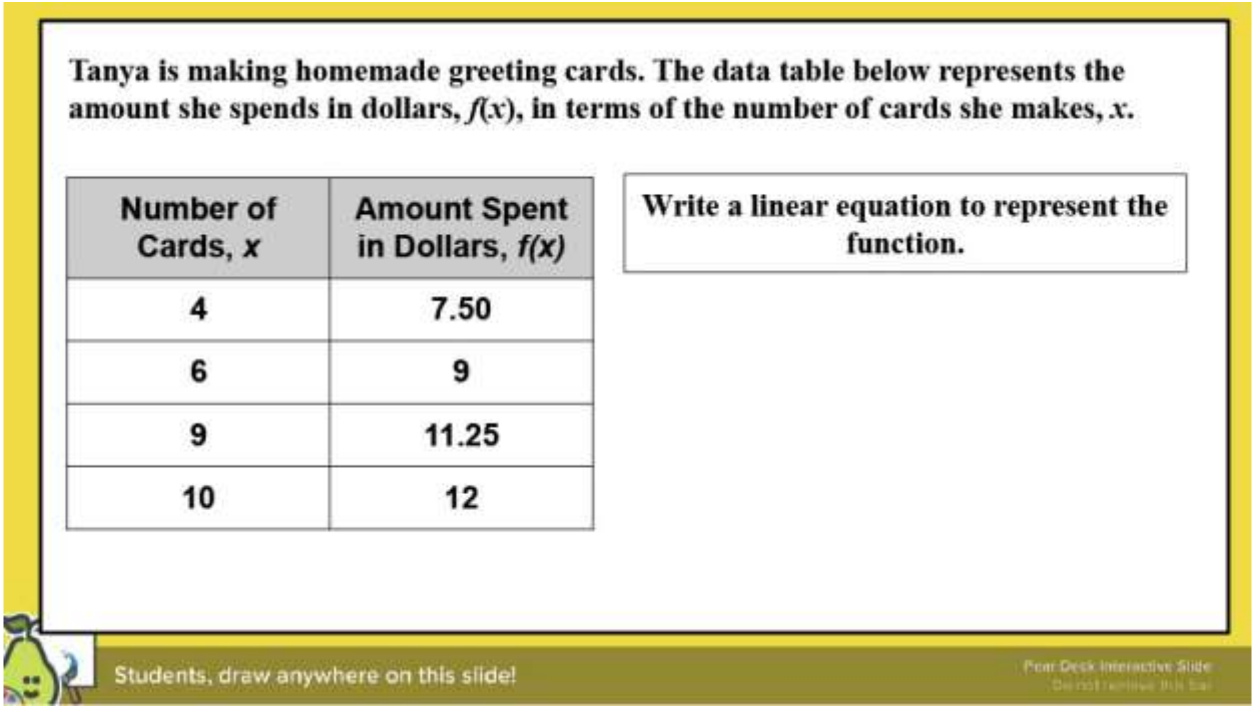
MATHEMATICAL TASK INTERVIEW – TASK AND PROBING QUESTIONS

TASK # 1


On the grid are 8 points from 2 **different** functions.
- 4 points fit a **linear** function
- The other 4 points fit a **non-linear function.**
Draw a line on the grid connecting the 4 points that would create a linear function.
y
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9
x
Students, draw anywhere on this slide!
Pear Deck Interactive Slide


Write a linear function that represents the line you drew connecting 4 of the points.
y
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9
x
Students, draw anywhere on this slide!
Pear Deck Interactive Slide

TASK # 2

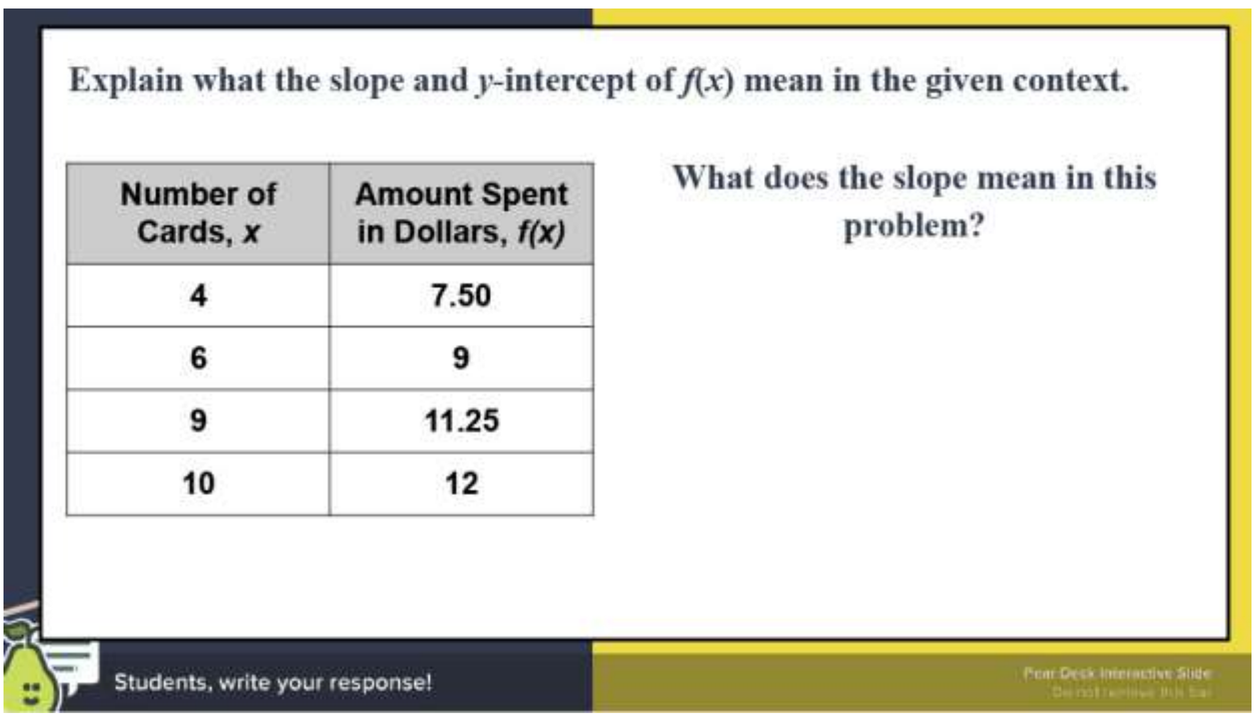


Tanya is making homemade greeting cards. The data table below represents the amount she spends in dollars, $f(x)$, in terms of the number of cards she makes, x.

Number of Cards, x	Amount Spent in Dollars, $f(x)$
4	7.50
6	9
9	11.25
10	12

Write a linear equation to represent the function.

Students, draw anywhere on this slide!

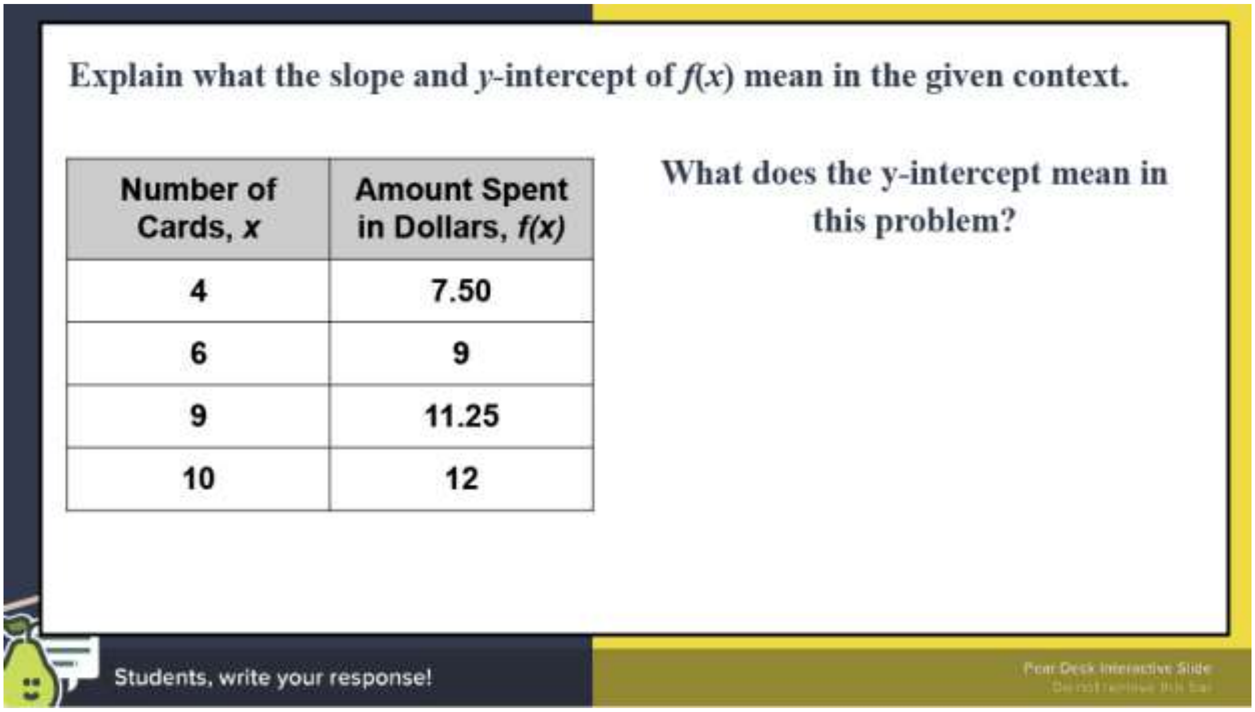


Explain what the slope and y-intercept of $f(x)$ mean in the given context.

Number of Cards, x	Amount Spent in Dollars, $f(x)$
4	7.50
6	9
9	11.25
10	12

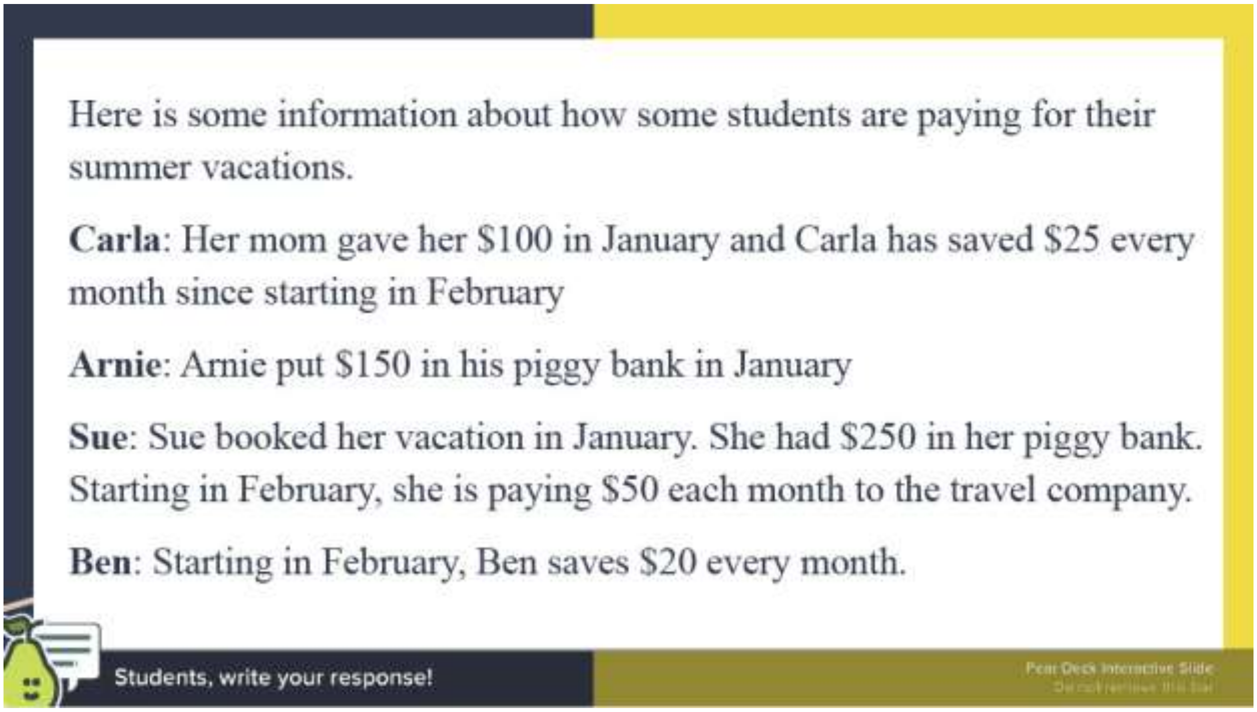
What does the slope mean in this problem?

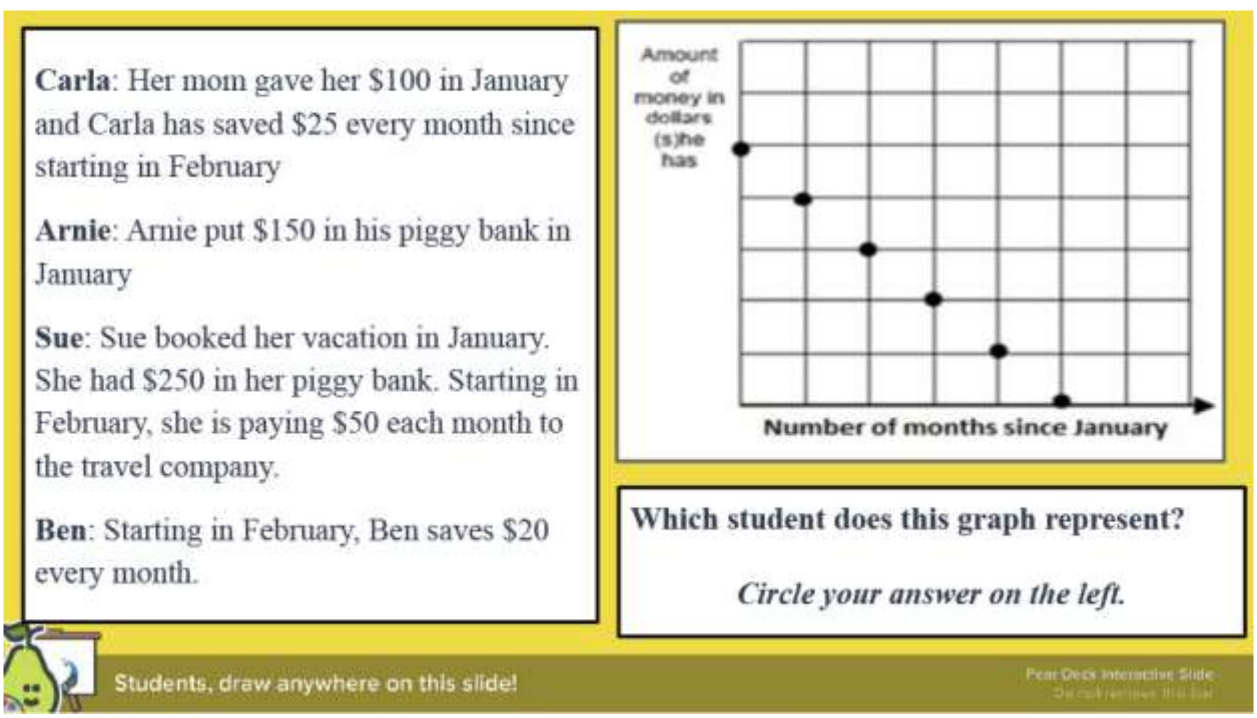
Students, write your response!

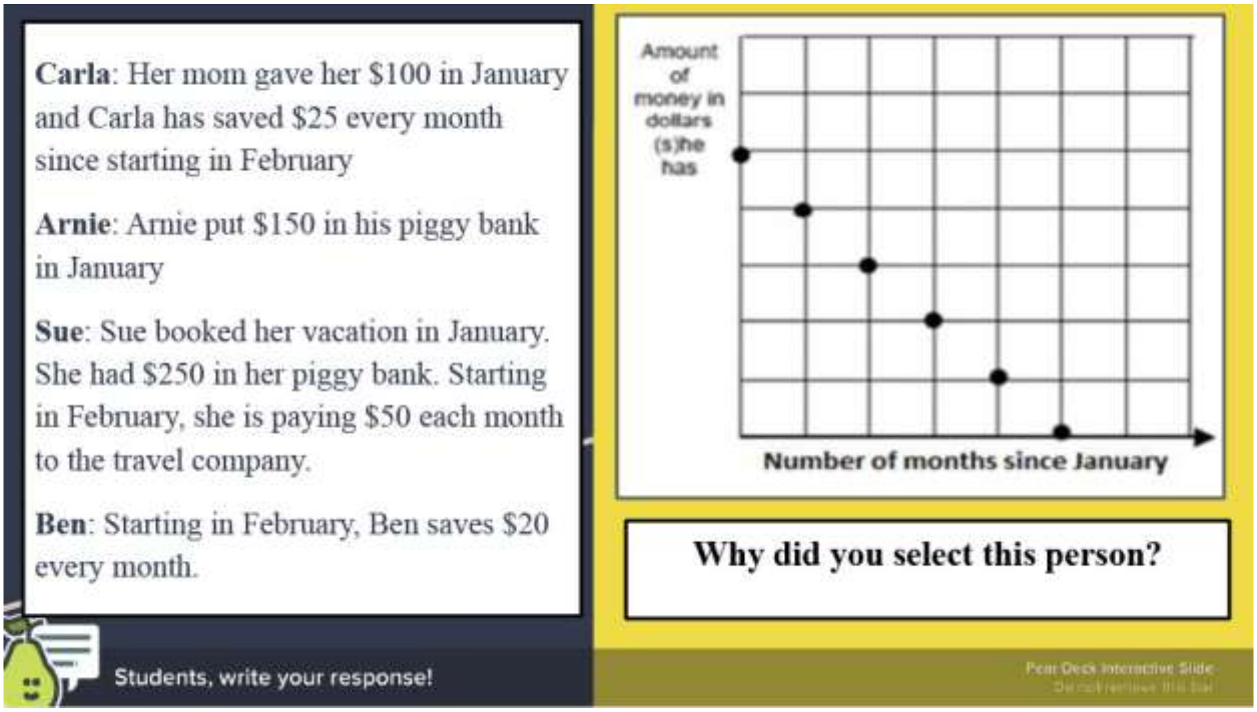


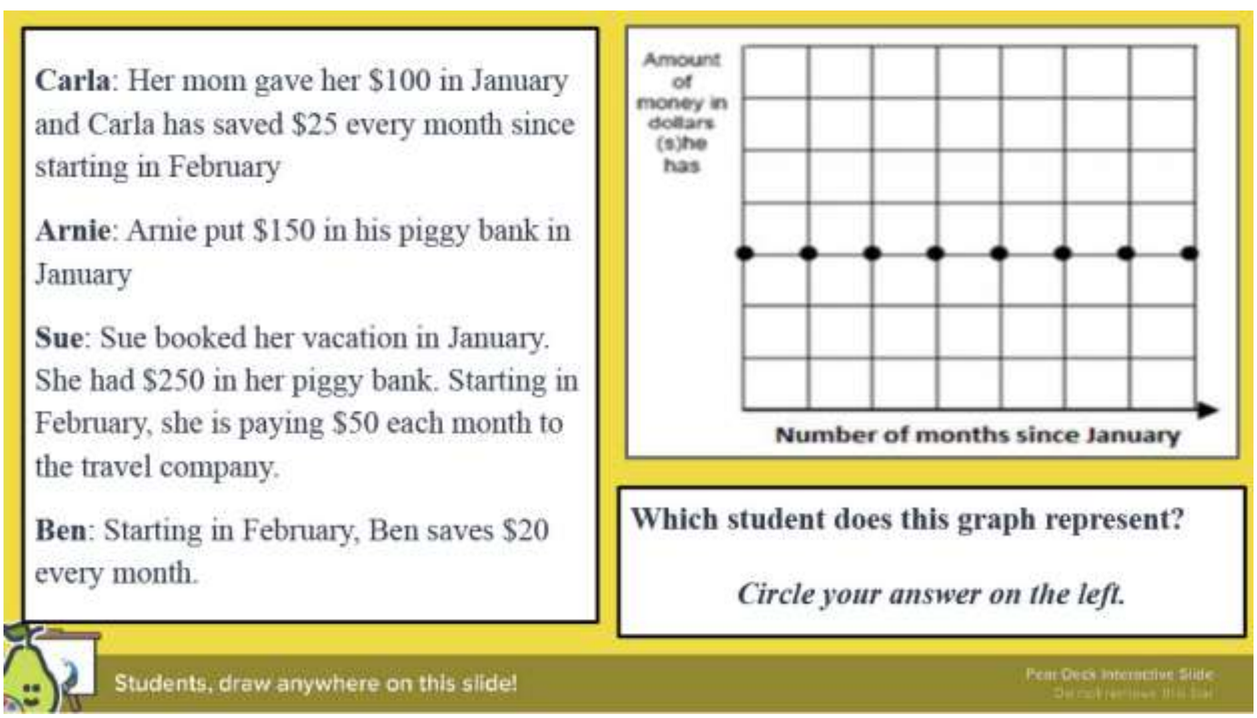
Number of Cards, *x*	Amount Spent in Dollars, *f(x)*
4	7.50
6	9
9	11.25
10	12

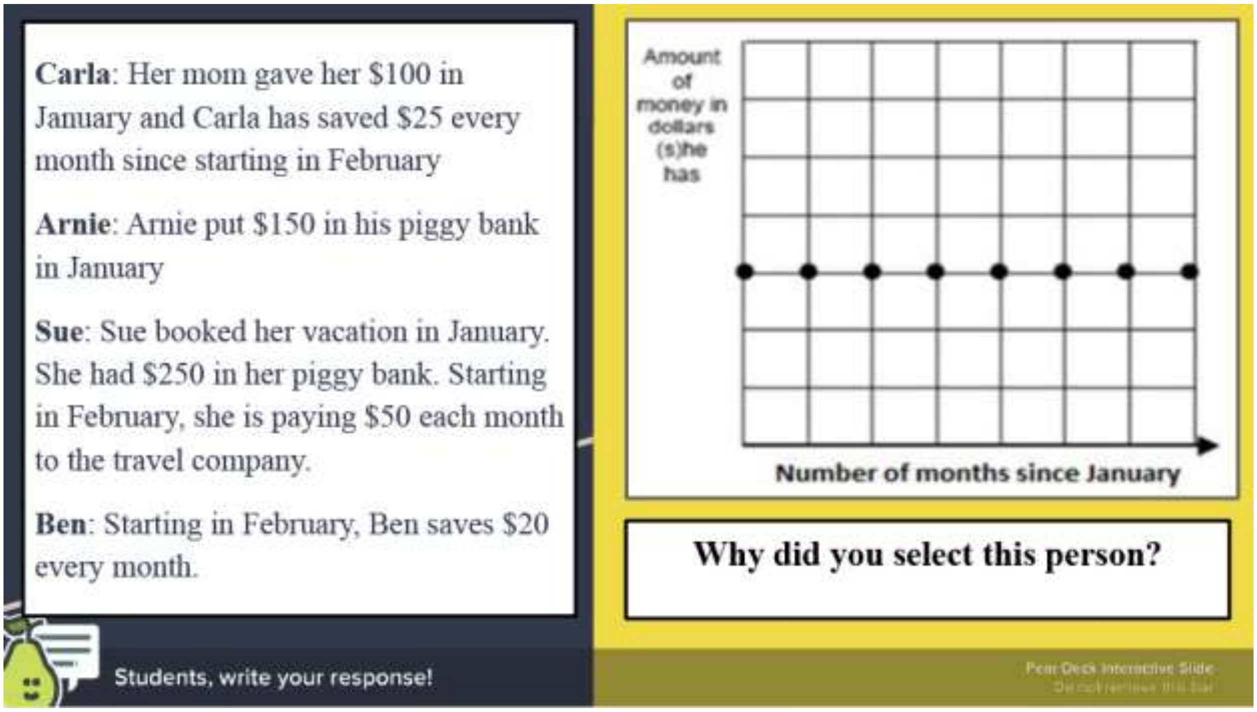
TASK #3

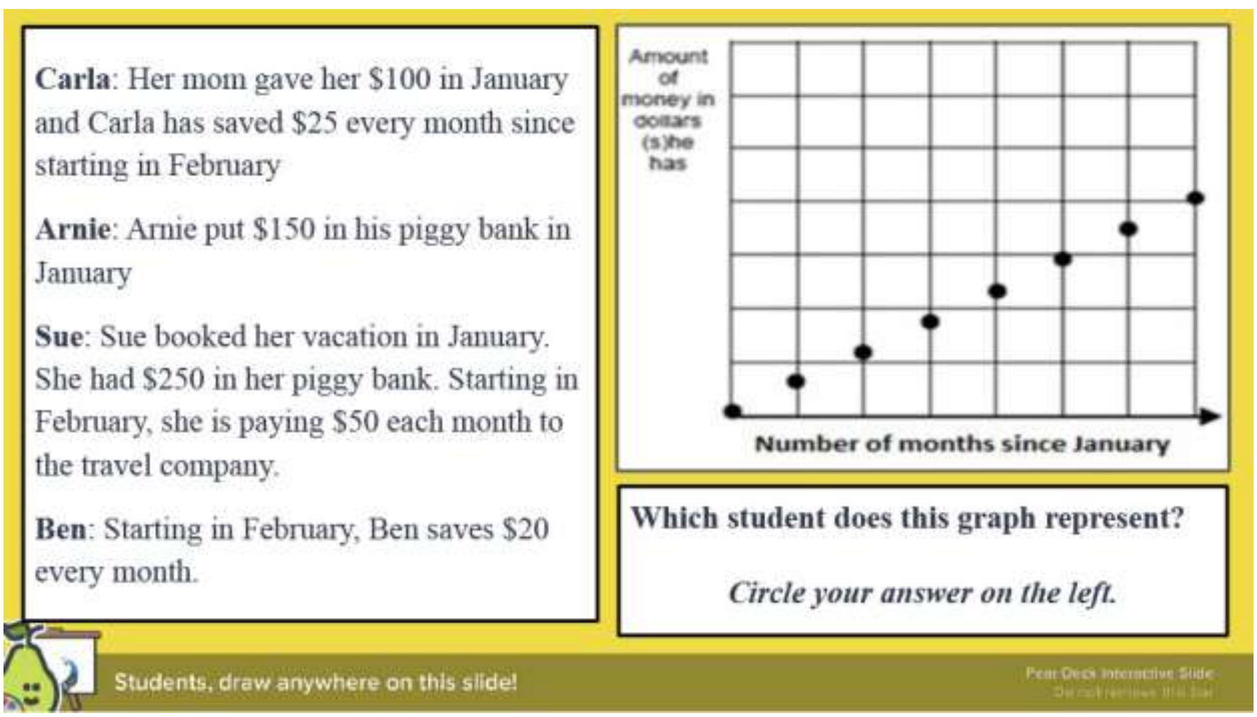

Here is some information about how some students are paying for their summer vacations.
Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Students, write your response!
Pear Deck Interactive Slide

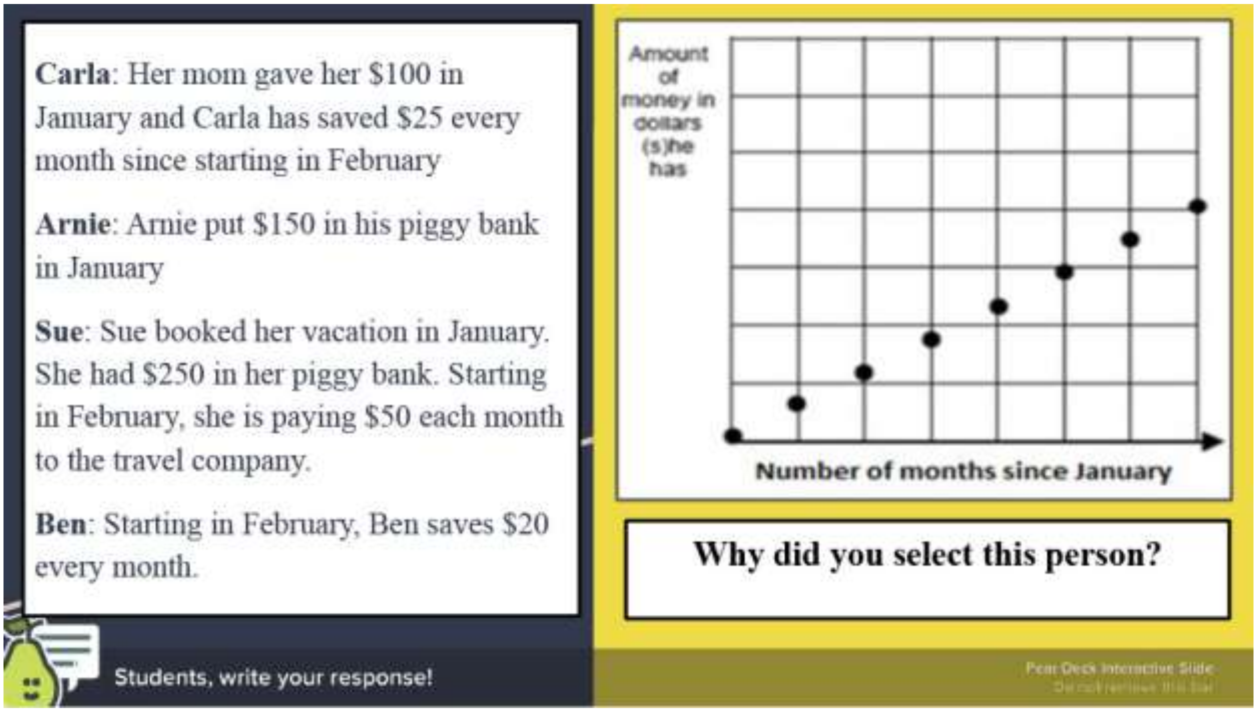

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Amount of money in dollars (s)he has
Number of months since January
Which student does this graph represent?
Circle your answer on the left.
Students, draw anywhere on this slide!
Pear Deck Interactive Slide

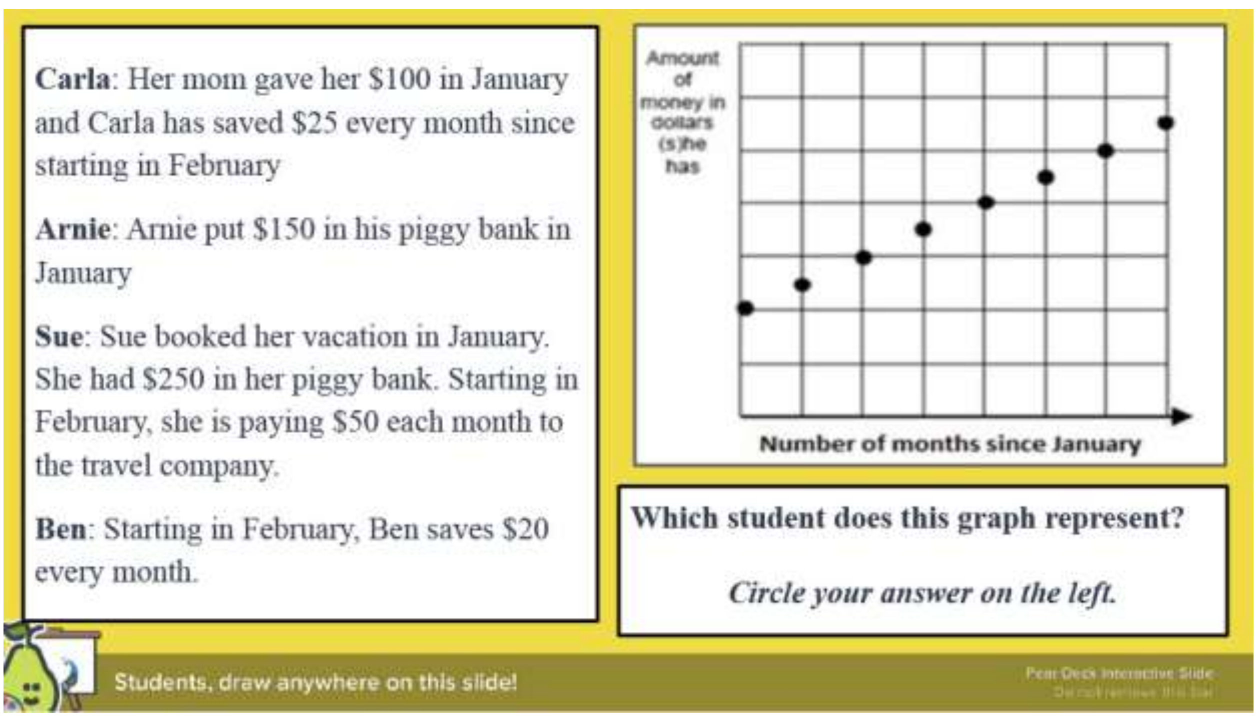

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Amount of money in dollars (s)he has
Number of months since January
Why did you select this person?
Students, write your response!

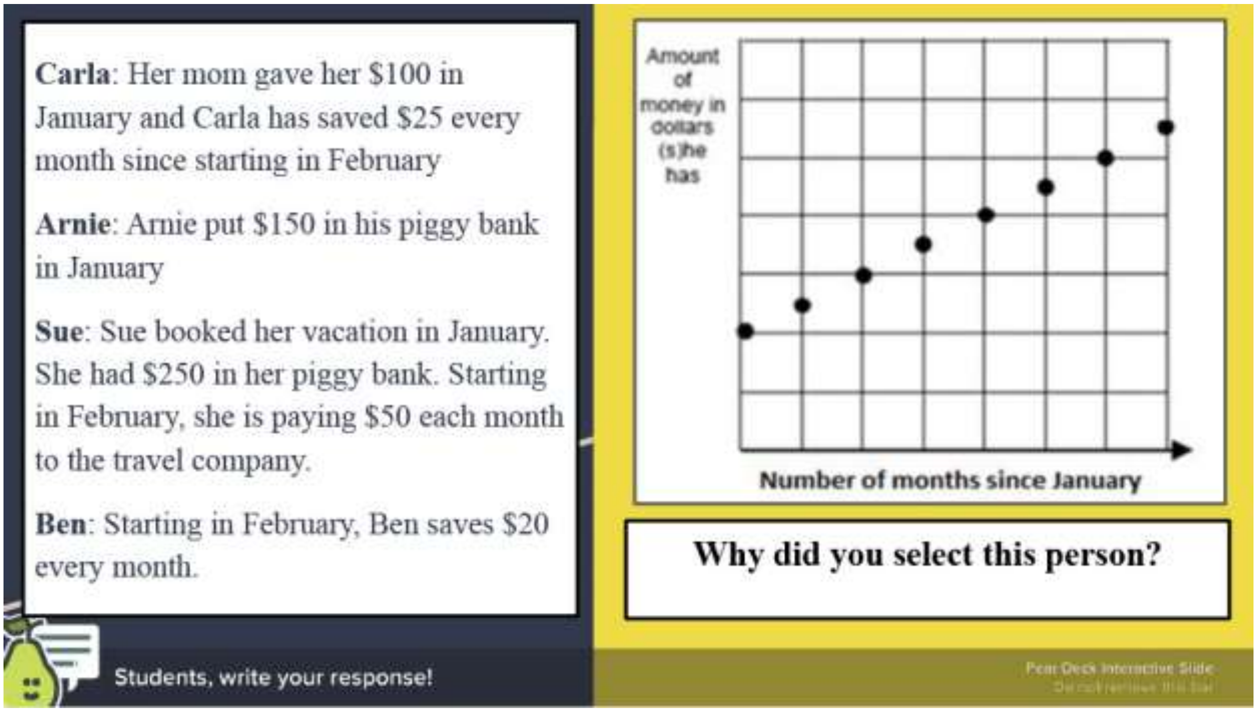

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Amount of money in dollars (s)he has
Number of months since January
Which student does this graph represent?
Circle your answer on the left.
Students, draw anywhere on this slide!

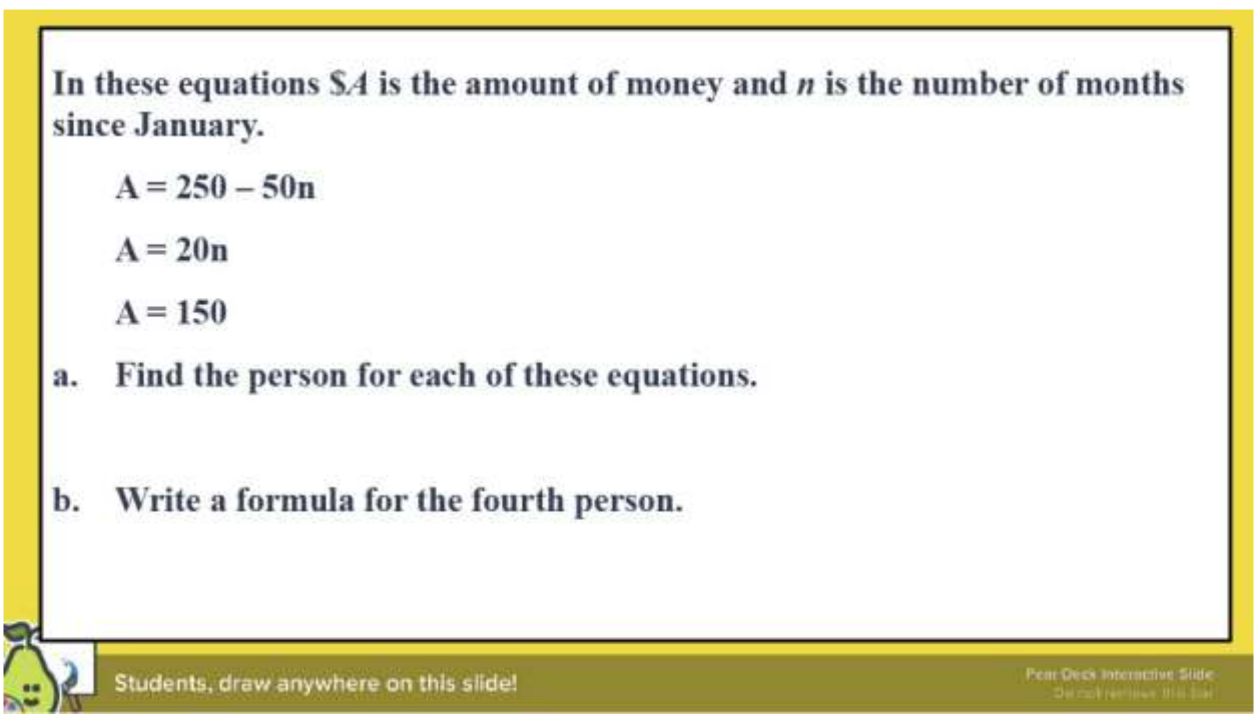

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Amount of money in dollars (s)he has
Number of months since January
Why did you select this person?
Students, write your response!
Pear Deck Interactive Slide

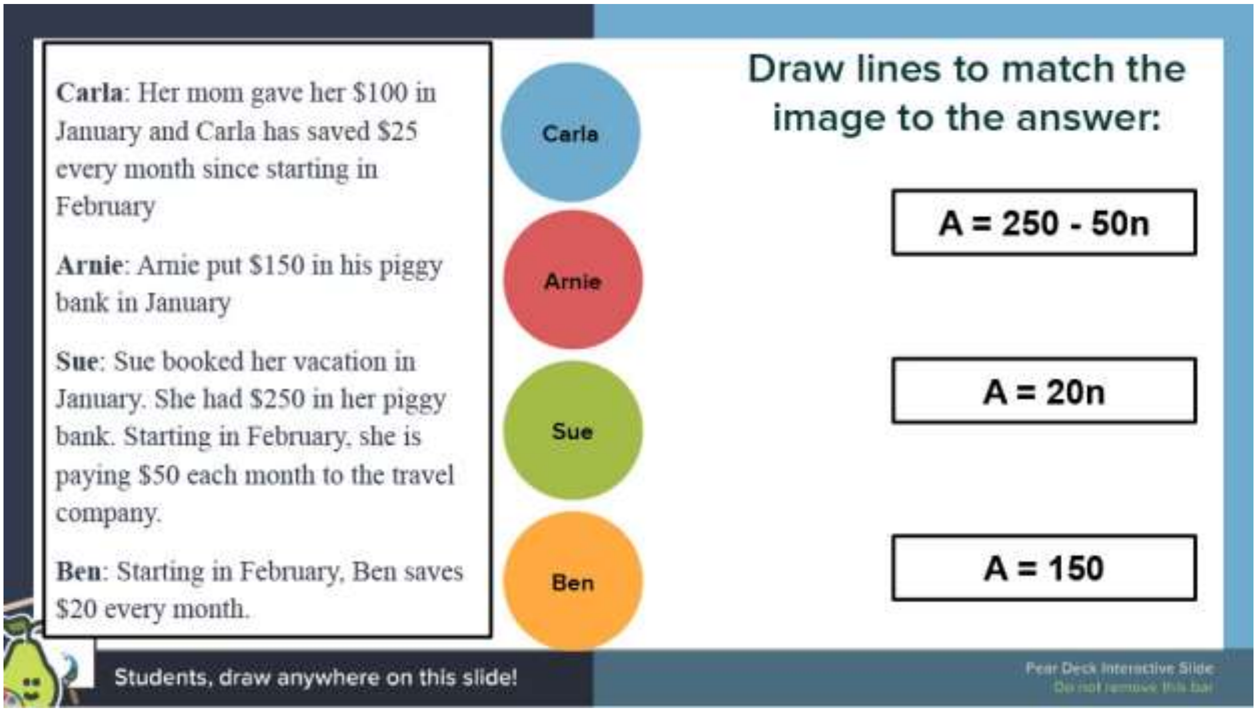

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Amount of money in dollars (s)he has
Number of months since January
Which student does this graph represent?
Circle your answer on the left.
Students, draw anywhere on this slide!
Pear Deck Interactive Slide

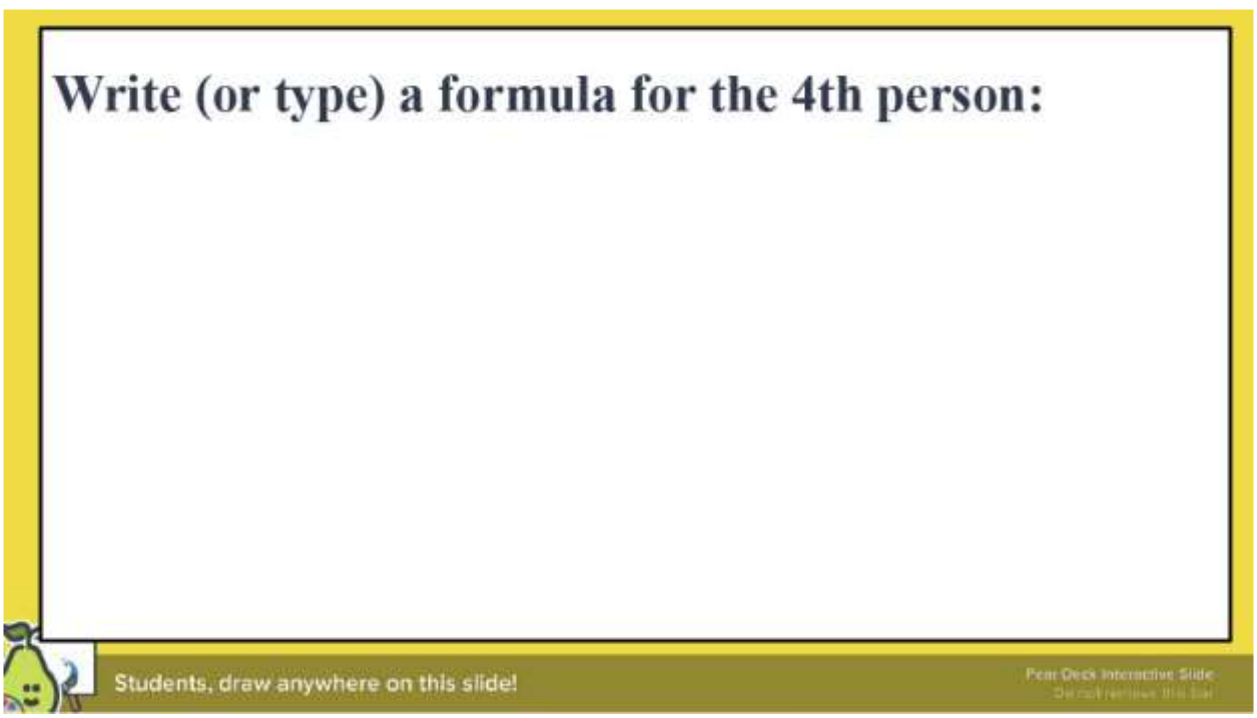

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Amount of money in dollars (s)he has
Number of months since January
Why did you select this person?
Students, write your response!

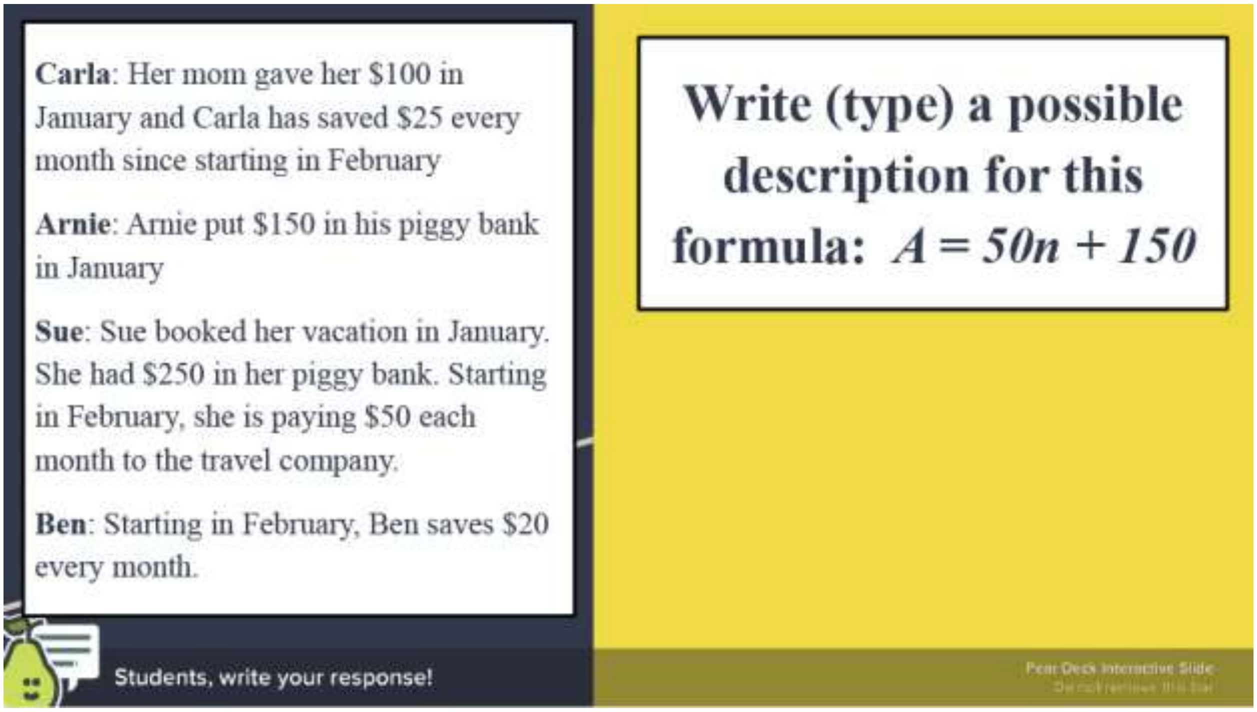

Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Amount of money in dollars (s)he has
Number of months since January
Which student does this graph represent?
Circle your answer on the left.
Students, draw anywhere on this slide!


Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Students, write your response!
Amount of money in dollars (s)he has
Number of months since January
Why did you select this person?
Pear Deck Interactive Slide


In these equations $A is the amount of money and n is the number of months since January.
A = 250 – 50n
A = 20n
A = 150
a. Find the person for each of these equations.
b. Write a formula for the fourth person.
Students, draw anywhere on this slide!
Pear Deck Interactive Slide


Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Carla
Arnie
Sue
Ben
Draw lines to match the image to the answer:
A = 250 - 50n
A = 20n
A = 150
Students, draw anywhere on this slide!
Pear Deck Interactive Slide


Write (or type) a formula for the 4th person:
Students, draw anywhere on this slide!
Pear Deck Interactive Slide


Carla: Her mom gave her $100 in January and Carla has saved $25 every month since starting in February
Arnie: Arnie put $150 in his piggy bank in January
Sue: Sue booked her vacation in January. She had $250 in her piggy bank. Starting in February, she is paying $50 each month to the travel company.
Ben: Starting in February, Ben saves $20 every month.
Write (type) a possible description for this formula: $A = 50n + 150$
Students, write your response!
Pear Deck Interactive Slide

Probing Questions

For each part of the task, the interviewer read the task aloud. The interviewer asked the participant to explain his/her/their work aloud while solving. The interviewer asked the following probing questions during the mathematical task:

1. Why did you write that?
2. How did you solve that?
3. How did you go about completing this problem?
4. What was the first thing you did? Why?
5. Please explain the meaning of your answer. What does your answer mean?
6. Did you use any strategies or tricks to remember this type of problem?
7. How did you know this graph matches with that person?
8. What is the slope? Rate of change? Tell me more about what you are thinking.
9. What is the *y*-intercept? How do you know? Tell me more about what you are thinking.
10. What is the meaning of the rate of change?
11. What is the meaning of the *y*-intercept?

APPENDIX G

ALIGNMENT OF MATHEMATICAL TASK INTERVIEW WITH LITERATURE

Alignment of Mathematical Task Interview with Literature

Big Idea	Task Number	Question
Rate of Change	1	Write an equation for the function. Show your work.
	1	What is the rate of change? How do you know?
	2	Explain what the rate of change of $f(x)$ means in the given context.
	3	Here are some illustrations. Match each person with a graph and explain how you decided
	3	Match the following 3 equations with the student.
Y-Intercept	1	Write an equation for the function. Show your work.
	1	What is the y-intercept? How do you know?
	2	Write a linear function $f(x)$ that represents the data
	2	Explain what the y-intercept of $f(x)$ means in the given context.
	3	Here are some illustrations. Match each person with a graph and explain how you decided
Multiple Representations	1	Write an equation for the function
	2	Write a linear function $f(x)$ that represents the data
	3	Here are some graphs illustrating these situations. Match each person with a graph and explain how you decided.
	3	In these equations, A is the amount of money and n is the number of months since January. Match the following 3 equations with the student.
	3	Write a possible description for this linear function: $A = 50n + 150$

APPENDIX H

SEMI-STRUCTURED INTERVIEW CODE LIST

Code List

Code	Data Exemplar
Breaking Down Content	*It was helpful that we had her break down the problem into steps for us to understand.*
Reference Points	*Maybe like posters in the room with some of the stuff that we have learned. Give me examples so I can look at the example and if I get stuck, I could look back.*
Guided Practice	*I like how she would do one problem together, and then we would do one problem on our own, and she would check it before we would go into groups or do our worksheet.*
Repeated Exposure	*I think the teacher has one type of problem and they break that problem down. But not just one time. They need to do it a few times and let us try it a few times.*
Multiple Ways to Solve	*I like when teachers explain it in so many different ways that all of us would understand it in our own unique way.*
Pacing	*They didn't give us enough time to finish our work.*
Re-explaining	*She will come to us when we do not understand and explain it again.*
Ownership of Learning	*Sometimes I wouldn't understand some of the stuff that they were saying, but I would like try to teach myself.*
Asking for Help	*If I still didn't get it, I would ask the teachers questions or ask for help*
Seeking Extra Help	*Even if I didn't understand it, they would always try to keep me after school or go to programs or come up at lunch. They were always there to help us if we needed it.*
Group Work	*Sometimes the teachers would break us into groups and one teacher would work with one group and another teacher would work with another group.*

APPENDIX I

MATHEMATICAL TASK INTERVIEW CODE LIST

Code List

Code	Data Exemplar
Sign of the Rate of Change	*Because she has 250 and the line was decreasing because she was paying, so it is negative.*
	There is no money putting in or taking out of the account. It is just staying the same.
	This graph is also increasing.
Procedural Approach	*So that would be 7 – 9 over 3 – 2. So that would be 7 minus 9 is -2 over 1, which is -2. M equals -2.*
	I counted the boxes.
	Because when I knew that 7.50 and 9 goes by 2.50.
Y-Intercept as a Separate Entity	*Because, well, it shows how much money she has in her account already and it shows how much money she is saving.*
	Because it is 100 in January and then she saved 25 each month.
	The y-intercept represents how much she spends in total.
Reliance on Algebraic Equations	*I would take an equation, and I would put it into the equation. I would take the (3, 7) and I would put the 3 in the spot of the x and the 7 in for y.*
	I think I can use substitution to do this using one of the points.
Straight Line	*Because this one wouldn't be in a straight line.*
	It makes a straight line.

APPENDIX J

MEMBER CHECK

Member Check

Q9 Thank you for participating in this research study. I would like to ask you a few questions about the patterns I found across all of the students that participated in this study.
This survey will take about 10 minutes. You do not have to answer any questions you do not wish to answer or are uncomfortable answering, and you may stop at any time. Your participation in this member check is completely voluntary.

We will make our best efforts to keep your answers confidential. No one except for the research team will have access to your answers.

If you have any questions, comments or concerns about the research, you can contact to one of the following researchers:
-Kayla Neill [redacted]
-Nicora Placa [redacted]

Additionally, if you have questions about your rights, or you have comments or concerns that you would like to discuss with someone other than the researchers, please call the CUNY Research Compliance Administrator at 646-664-8918. Alternately, you can write to:

CUNY Office of the Vice Chancellor for Research
Attn: Research Compliance Administrator
205 East 42nd Street
New York, NY 10017

○ I consent to participating in the member check (1)

○ I do not consent to participating in the member check (2)

Skip To: End of Survey If Thank you for participating in this research study. I would like to ask you a few questions about... = I do not consent to participating in the member check

Page Break

Q10

Mathematical Task Interview Findings:

1. Students used a procedural approach the most such as the slope-formula or counted boxes on the graph to find the rate of change.

2. Students preferred to use an algebraic equation and procedure to find the y-intercept (substitution using one coordinate pair).

3. Students could match a real-world description with a graphical representation and algebraic equation.

4. Students knew that the sign of the rate of change is important.

5. Students had a difficult time explaining the meaning of the rate of change and y-intercept in terms of the amount it cost Tanya to make each greeting card.

ICT Mathematics Instruction Interview Findings:

1. Students prefer their teachers to break down mathematics instruction and provide several opportunities for students to practice together with the teacher.

2. Students like when teachers show them multiple ways to solve a problem and when teachers allow them to use the method that works best for them.

3. For the most part, students think that the speed of mathematics class is too fast. Students do not always have enough time to follow along, ask questions, and complete their work.

4. Students like when their teachers allow them to work in groups, so that they can share their understanding and learn from their peers. However, not all of their teachers give them time to work in groups.

5. Students seek outside help from their teacher or other teachers during their free time such as at lunch or after school.

Conclusions

Students show an understanding of linear functions. Students can match real-world descriptions with a graph and an algebraic equation. Students can write an equation based on a given description. Students know when and how to use the slope-formula to find the rate of change. Students know when to use substitution to find the y-intercept. Students can identify the rate of change and y-intercept and write an algebraic equation in slope-intercept form. However, students have a more difficult time trying to explain what the rate of change and the y-intercept mean.

Students prefer direct instruction in mathematics. This is when the teacher breaks down a concept several times for students. Students like to follow along as the teacher breaks it down. They prefer when the teachers goes over it a few times so that they can understand it before trying to do the math on their own. Students shared that they are aware that all students learn differently. Because of this, they want their teachers to show them more than 1 way to solve a problem. They also like when their teachers allow them to use whichever method works best for them to solve. They do not like when their teachers only show them 1 method to solve and force them all to use that 1 method. Students feel that the speed of the class is too fast. The teacher moves on to the next problem and next topic before students feel comfortable with the work. Students like to work in groups so that they can talk to their classmates about the work and learn from each other.

Suggestions for Mathematics Teachers

Mathematics teachers should break down new mathematics problems for students. They should go over a few of the same kinds of problems before asking students to complete the work on their own. Mathematics teachers should also try to explain the new math topic in a few different ways because not all students think the same. A method that works for 1 students may not work for another student. Students like to hear different methods and they like to use the one that they like the most to complete their work. Teachers should give them this option. Teachers should also slow down the pace of instruction. Students want to feel more comfortable with the math work before the teacher moves on to another problem or another topic. Teachers should check in with students to make sure they understand the problem and are ready to move on to the next. Teachers should also include more group work. Students like to work in groups because they like to share their thoughts and hear how their classmates did the work. Students feel that they can learn from each other. They also like when the teacher will come over to them in their small groups to re-explain something or to answer questions.

Page Break

Q1 Do the findings accurately describe what was discussed during your mathematical task interview?

- ◯ Yes (1)
- ◯ No (2)

Skip To: Q2 If Do the findings accurately describe what was discussed during your mathematical task interview? = Yes

Q7 If you selected "no," please explain why.

____

Page Break

Q2 Do the findings accurately describe what was discussed during your interview about your thoughts and feelings of your mathematics instruction?

- ◯ Yes (1)
- ◯ No (2)
- ◯ Sort Of (3)

Skip To: Q3 If Do the findings accurately describe what was discussed during your interview about your thoughts... = Yes

Q8 If you selected "sort of" or if you selected "no," please explain why.

____

Page Break

Q3 Do you think the conclusions make sense?

- ◯ Strongly agree (1)
- ◯ Agree (2)
- ◯ Somewhat agree (3)
- ◯ Neither agree nor disagree (4)
- ◯ Somewhat disagree (5)
- ◯ Disagree (6)
- ◯ Strongly disagree (7)

Q4 Do you think the suggestions for math teachers of students with Learning Disabilities make sense? Why or why not?

____

Q5 What other thoughts do you have about the findings?

____

Q6 Do you have any other comments?

____

Page Break

Q11 Thank you for participating in this member check. If you have questions, please contact either researcher or CUNY:

-Kayla Neill [redacted]
-Nicora Placa [redacted]
or CUNY Research Compliance Administrator at 646-664-8918.

Alternately, you can write to:

CUNY Office of the Vice Chancellor for Research
Attn: Research Compliance Administrator
205 East 42nd Street
New York, NY 10017

End of Block: Default Question Block
